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NON-LOGICAL CONSEQUENCE

Abstract: Contemporary philosophers generally conceive of consequence as necessary truth-preservation. They generally construe this necessity as logical, and operationalize it in substitutional, formal or model-theoretic terms as the absence of a counter-example. A minority tradition allows for grounding truth-preservation also on non-logical necessities, especially on the semantics of extra-logical constants. The present article reviews and updates the author's previous proposals to modify the received conception of consequence so as to require truth-preservation to be non-trivial (i.e. not a mere consequence of a necessarily true implicatum or a necessarily untrue implicans) and to allow variants of the substitutional, formal and model-theoretic realizations of the received conception where the condition underwriting truth-preservation is not purely formal. Indeed, the condition may be contingent rather than necessary. Allowing contingent non-trivial truth-preservation as a consequence relation fits our inferential practices, but turns out to be subject to counter-examples. We are left with an unhappy choice between an overly strict requirement that non-trivial truth-preservation be underwritten by a necessary truth and an overly loose recognition of non-trivial truth-preservation wherever some truth underwrites it. We need to look for a principled intermediate position between these alternatives.

Keywords: consequence, logical consequence, non-logical consequence, Alfred Tarski, truth-preservation, necessity, substitutional, formal, model-theoretic

1. Consequence in contemporary philosophy

In contemporary philosophy, consequence is generally construed as necessary truth-preservation. A conclusion is said to follow from the premiss(es) from which it is drawn if and only if it is necessary that, if the premiss or premisses are true, then the conclusion is also true (Tarski 2002/1936, pp. 178, 183–184; Salmon 1963, p. 18; Etchemendy 1990, pp. 81–82; Forbes 1994, p. 3, Copi & Cohen 2001, p. 43; Hurley 2006, p. 41; Jeffrey 2006, p. 1). Equivalently, it is impossible for the conclusion to be untrue while the premiss(es) are true.[1]

[1] I write 'untrue' rather than 'false', in order to leave open the possibility of a conclusion's being neither true nor false.

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The impossibility in question is most commonly construed as logical or formal, meaning that the form of the conclusion and the premiss(es) rules out the combination of an untrue conclusion with true premiss(es). The application of this conception to reasoning and arguments in a natural language requires that the sentences of the language be regimented into a "canonical notation" (Quine 1960), which can then if desired be recast in a formal language whose extra-logical constants may be in themselves uninterpreted. For example, the logical operation of conjunction, indicated in English by the word 'and', is commutative, in the sense that, from the conjunction of one sentence with a second sentence, there follows the conjunction of the second sentence with the first.[2] Thus, from 'Snow is white and grass is green' there follows 'Grass is green and snow is white'. But one cannot apply this principle directly to all English-language sentences in which the word 'and' is the main connective joining two clauses. To take a mildly scatological example, the situation in which a man pulls down his pants and pees is different from the situation in which he pees and pulls down his pants. What needs to be made explicit in regimenting the sentence 'he pulled down his pants and peed' is that in English a sequence of two tensed clauses joined by 'and' implicitly claims that the event or state of affairs described in the first-mentioned clause precedes the event or state of affairs described in the second-mentioned clause. In canonical notation, the sentence might be recast as follows: In some time interval k before now he pulls down his pants, and in some time interval l before now he pees, and k precedes l. With this explicitation, any sentence obtained by permutation of the clauses of the recast sentence that are joined by 'and' follows from it.

2. Tarski's conception of consequence

Although the necessity in the condition of necessary truth-preservation is most commonly construed as logical necessity, there is a minority philosophical tradition – whose members include Bolzano (1972/1837), Peirce (1955/1877), Sellars (1953), Ryle (1960/1954), Toulmin (1958), George (1972, 1983), and Brandom (1994, pp. 97–104; 2000, pp. 52–55) – that construes it as including other kinds of necessity as well. Perhaps surprisingly,

[2] For simplicity, I am taking sentences to be the relata of the consequence relation. Nothing in this article should depend on this decision. The same points about consequence could be made if one takes entities other than sentences to be the primary truth-bearers – e.g. utterances, statements or propositions.

Alfred Tarski can be counted as a member of this tradition. In his classic paper "On the concept of following logically", Tarski lays down the following necessary condition (F) for the material adequacy of an account of what it is for a sentence X to follow logically from the sentences of a class $\mathcal{K}$:

(F) *If in the sentences of the class $\mathcal{K}$ and in the sentence X we replace the constant terms which are not general-logical terms correspondingly by arbitrary other constant terms (where we replace equiform constants everywhere by equiform constants) and in this way we obtain a new class of sentences $\mathcal{K}'$ and a new sentence X', then the sentence X' must be true if only all sentences of the class $\mathcal{K}'$ are true.* (Tarski 2002/1936, pp. 183–184; italics in original)[3]

Tarski's condition (F) in fact combines two conditions, which he articulates separately before stating condition (F). The first condition is the condition of necessary truth-preservation, stated quite generally in a way that does not restrict consequence to logical consequence:

> ...it cannot happen that all the sentences of the class $\mathcal{K}$ would be true but at the same time the sentence X would be false. (Tarski 2002/1936, p. 183)

This condition combines an impossibility condition ("cannot") with a co-temporality condition ("at the same time"). Tarski does not explain what he means by either of these conditions. Given that Tarski's focus was on deductive mathematical theories, whose sentences do not change their truth-value over time, the co-temporality condition "at the same time" is most plausibly construed as a metaphor for co-situatedness, "in the same situation" or "in the same circumstances". That is, Tarski is claiming that what cannot happen when a sentence X is a consequence of all the sentences of a class $\mathcal{K}$ is that, given one and the same situation, all the sentences of the class $\mathcal{K}$ are true but the sentence X is false. As for the impossibility condition, I have argued (in Tarski 2002, pp. 168–170) that it is the condition that there are no circumstances in which both the implying sentences are true and the implied sentence false. On this interpretation, the impossibility condition and the co-temporality condition are the same condition: that there is no

[3] Here and elsewhere, I use the exact translation into English by Magda Stroińska and myself of the Polish version of Tarski's paper (Tarski 1936a), which I argued in (Tarski 2002) is more authoritative than the German version, also written by Tarski (Tarski 1936b), which was used as the basis of the previous rather inexact translation of the paper into English (Tarski 1956 and 1983, pp. 409–423).

(possible) situation in which all the sentences of the class $\mathcal{K}$ are true but the sentence X is false.

Tarski immediately follows his statement of the requirement of necessary truth-preservation with an argument for the following additional requirement for a specifically logical consequence relation:

> ... following ... cannot be lost as a result of our replacing the names of ... objects in the sentences under consideration by names of other objects (Tarski 2002/1936, p. 183).

As is well known, Tarski argued that this substitutional condition, although necessary for logical consequence, is insufficient, because a language might lack names for the objects that would constitute a counter-example when a sentence X does not follow logically from the sentences of a class $\mathcal{K}$. He therefore proposed what became the contemporary model-theoretic conception of logical consequence:

> *We say that the sentence X f o l l o w s l o g i c a l l y from the sentences of the class $\mathcal{K}$ if and only if every model of the class $\mathcal{K}$ is at the same time a model of the sentence X.* (Tarski 2002/1936a, p. 186; italics and extra spaces in the original)

In this definition, Tarski meant by a model a sequence of objects that satisfies a sentential function, a rather different conception than the contemporary notion of a model as an interpretation. In contemporary work in formal logic, formal languages are usually constructed with a distinction between interpreted logical constants (such as the signs signifying logical conjunction and universal quantification) and uninterpreted extra-logical constants. The semantics for such a language specifies what constitutes an interpretation of sentences in the language, which typically includes specification of a domain or "universe of discourse" (a non-empty set of objects) and an assignment to each extra-logical constant of some object defined in terms of the domain (a member of the domain, a subset of the domain, a set of ordered pairs of members of the domain, etc.). A sentence X of a language is said to follow logically from the sentences of some class $\mathcal{K}$ of sentences of the language if and only if every true interpretation of the sentences of the class $\mathcal{K}$ is also a true interpretation of the sentence X. Although not identical to Tarski's conception, this definition captures its spirit in the contemporary framework for formal work.

In his article, Tarski pointed out quite rightly that the scope of logical consequence as thus defined depends on how one divides logical terms from extra-logical terms. In a substitutional conception of logical consequence,

the logical terms are those not subject to substitution when searching for a substitutional counter-example – i.e. a parallel argument with true premisses and an untrue conclusion, obtained by uniform substitution on the original argument's extra-logical terms. In a model-theoretic conception of logical consequence, the logical terms are those not needing interpretation in the search for a model-theoretic counter-example – i.e. an interpretation in which the premisses of the argument are true but its conclusion untrue. If all terms are extra-logical, then on either the substitutional or the model-theoretic conception any sentence is a logical consequence only of itself (either alone or in combination with other sentences). If all terms are logical, then on the substitutional conception any true sentence is a logical consequence of any sentences and any sentence is a logical consequence of any class of sentences that are not all true. That is, logical consequence reduces to the so-called "material implication" of medieval logicians (*consequentia materialis*), a relation that holds in all cases except those in which the sentences of the class $\mathcal{K}$ are true but the sentence X is untrue. Tarski made the same claim about his version of the model-theoretic conception (2002/1936, pp. 188–189). I argued in (Tarski 2002, p. 171) that Tarski's claim can be defended if we suppose that the domain for the language is fixed, as Tarski's article implicitly assumed. On the contemporary model-theoretic conception of logical consequence, however, the domain with respect to which sentences of a formal language are interpreted is not fixed, and so fixing the interpretation of all terms does not reduce logical consequence to material implication. For example, the sentence 'There are at least two objects' does not follow from the sentence 'there is at least one object', since the first sentence is false but the second sentence true when the domain consists of just one object. Nevertheless, treating all terms as logical, in the sense that their interpretation is fixed for each possible size of the domain (e.g. from one object to denumerably many objects) greatly expands the extension of the concept of logical consequence.

Between the extreme that narrows the extension of logical consequence so that any sentence is a logical consequence only of itself, and the extreme that makes it coextensive with material implication, many intermediate positions are possible. In his 1936 article, Tarski confessed ignorance of any objective basis for dividing logical from extra-logical terms (2002/1936, p. 188), i.e. for selecting a particular intermediate position between the extremes just mentioned. Tarski speculated that no such objective basis might be found, in which case the model-theoretic conception of logical consequence would be relative to a definite but somewhat arbitrary division of the terms of a language into logical and extra-logical terms (pp. 189–190).

In his condition (F), Tarski referred to logical terms as "general-logical terms", a locution apparently reflecting his belief at the time that the logical terms are those that occur in all axiomatized deductive theories and in everyday life, whereas extra-logical terms are "specifically metalinguistic" or "specifically mathematical" (Tarski 2002, pp. 161–162). In later work (Tarski 1986/1966), Tarski proposed that the logical terms are those denoting notions that are invariant under all transformations of a domain into itself. For example, no name of an individual object in a domain is a logical term, because one can always transform any domain into itself in such a way that an arbitrarily selected individual member of it becomes another individual object. On the other hand, terms signifying the universe class and the empty class are logical, because their denotation remains the same under any transformation of any given domain into itself. Other logical "notions", as Tarski calls the denotata of logical terms, are the relations of identity and non-identity between individuals, the cardinality of classes of individuals, and the relations of inclusion, disjointness and overlap between classes (Tarski 1986/1966, pp. 150–151). But the criterion of invariance under transformations of a domain into itself, objective as it is, allows for some terms to be logical terms in one language but extra-logical in another. As Tarski points out (1986/1966, pp. 152–153), set membership is a logical notion if set theory is constructed in the fashion of Whitehead and Russell's *Principia Mathematica* via a higher-order logic involving a theory of types, but a non-logical notion if set theory is constructed in the fashion of Zermelo in a first-order logic in which a single domain includes individuals, classes of individuals, classes of classes of individuals, and so on. The ambiguous status of set membership leaves indeterminate the question of whether mathematical notions are logical notions, since set theory is basic to mathematics and all the notions of set theory can be defined in terms of set membership with the help of logical notions.

3. Extending Tarski's condition (F) to non-logical consequence

However the line is drawn between the logical and the extra-logical terms of a language, one can modify Tarski's condition (F) so as to permit some extra-logical terms to be treated as if they were logical. That is, in the search for a counter-example, these terms would not be subject to substitution (on a substitutional approach) or to variant interpretations (on a model-theoretic approach). The revised condition (F′) might be written as follows:

(F′) If in the sentences of the class $\mathcal{K}$ and in the sentence X we replace some or all of the constant terms which are not logical terms correspondingly by arbitrary other constant terms (where we replace equiform constants everywhere by equiform constants) and in this way we obtain a new class of sentences $\mathcal{K}'$ and a new sentence X', then the sentence X' must be true if only all sentences of the class $\mathcal{K}'$ are true.

In a more contemporary idiom, we might phrase the condition as follows:

(F″) For some non-empty subset of the extra-logical constants in the sentences of the class $\mathcal{K}$ and in the sentence X, if uniform substitution on these constants produces a new class of sentences $\mathcal{K}'$ and a new sentence X', then the sentence X' must be true if all the sentences of the class $\mathcal{K}'$ are true.

The requirement that the set of substitutable extra-logical constants be non-empty is meant to rule out treating the "material implication" of medieval logicians as a consequence relation. Intuitively, the mere fact that it is not the case that all the sentences of the class $\mathcal{K}$ are true and the sentence X is untrue does not suffice to make X follow, even non-logically, from the sentences of the class $\mathcal{K}$; for example, nobody would suppose that 'grass is green' follows from 'snow is white'.

The revised condition (F″) is a generalization of Tarski's condition (F), which then becomes the special case in which necessary truth-preservation continues to hold when substitution is allowed on the entire set of extra-logical constants. In general, however, application of (F″) would require multiple tests to see whether an argument met it. Take the hackneyed standard philosopher's example, 'Socrates is human, so Socrates is mortal', which we might put into a regimented language as 'HUMAN(Socrates), so MORTAL(Socrates)', where the extra-logical constants are the one-place predicates 'HUMAN' and 'MORTAL' and the name 'Socrates' and there are no logical constants. We have seven non-empty subsets of these extra-logical terms with respect to which condition (F″) might be met: {'HUMAN'}, {'MORTAL'}, {'Socrates'}, {'HUMAN', 'MORTAL'}, {'HUMAN' 'Socrates'}, {'MORTAL', 'Socrates'} and {'HUMAN', 'MORTAL', 'Socrates'}. Treating condition (F) as a necessary condition for logical consequence, we find that the conclusion that Socrates is mortal is clearly not a logical consequence of the premiss that Socrates is human, since condition (F″) is not met when all the extra-logical constants are subject to substitution, i.e. with respect to {'HUMAN', 'MORTAL', 'Socrates'}. In particular, substitution of 'PERSIAN' for 'MORTAL' produces an argu-

ment in which the premiss is true but the conclusion false, showing that the conclusion of this parallel argument need not be true when its premiss is true (since what is actually not the case is not necessarily the case). The same counter-example shows that the conclusion is not a consequence of the premiss with respect to any set of extra-logical constants that includes the predicate 'MORTAL'. On the other hand, since the conclusion is true, no substitution solely for 'HUMAN' will produce a parallel argument with a true premiss and an untrue conclusion, so that we cannot so easily show that the conclusion does not follow with respect to the set {'HUMAN'}. Here one needs to fall back on intuitive judgments of necessity, looking for a substitution for 'HUMAN' on which the premiss, though true, clearly does not necessitate the truth of the conclusion. For example, it is true that Socrates weighs more than a kilogram, but intuitively it is not necessary that Socrates is mortal if Socrates weighs more than a kilogram. The name 'Socrates' might for example refer to a large boulder, one that weighs more than a kilogram, but boulders are not mortal: since they are never alive, they never die. A similar reflection shows that the conclusion does not follow with respect to the set {'HUMAN' 'Socrates'}, since we can substitute a name of the aforesaid boulder for the name 'Socrates'. The remaining option is that the conclusion follows with respect to the set {'Socrates'}. Here we find not only that no substitution on 'Socrates' produces an argument with a true premiss and an untrue conclusion, but also that it is plausible to hold, in a way that it was not when we substituted for 'HUMAN', that, if the parallel argument has a true premiss then the conclusion must be true. That is, it is not just true as a matter of fact that, if someone is human, that individual is mortal, but it is a matter of necessity. Intuitively, this necessity is not logical, since there is no specifically logical inconsistency in supposing that a particular human being is immortal. Nor does the necessity seem semantic, since the postulation of an immortal human being, say in a work of science fiction, does not seem to involve a confusion about the meaning of terms, in contrast to the way in which the postulation of a married bachelor would involve semantic confusion. Rather, the necessity seems to be physical, or more specifically physiological. Human beings inevitably undergo a process of aging that eventually results in death due to failure of one or more of their life-support systems (circulatory, respiratory, excretory, etc.) if they do not die earlier from some other cause.[4]

[4] At least, so we suppose. Research on aging may lead to techniques of preventing human aging, in which case human immortality would become physiologically possible. But the "may" here is epistemic. At the moment, as far as we know, it is physiologically inevitable that every human dies.

The need to appeal in applications of condition (F″) to intuitive judgments of necessity is a weakness, since one person may come to a different judgment than another as to whether a particular conditional is necessary. Without stated criteria of necessity, it is impossible to resolve such differences of intuitive judgment rationally, except by inviting the disputants to reconsider their judgments or to take notice of the considered judgments of others about the necessity of the conditional in question. Tarski solved this problem by abandoning the requirement that truth-preservation be necessary. His model-theoretic conception of logical consequence simply specifies that every model (i.e. true interpretation) of the input sentences is also a model (true interpretation) of their logical consequence. Interpretations are constructed with reference to the world as it is, not to the world as it might be.

In *The Concept of Logical Consequence* (1990), John Etchemendy objected that Tarski's reduction of logical consequence to the simple truth of a universal generalization both undergenerates and overgenerates consequences. Even where it gives the right result, he claimed, it does so for the wrong reason. Etchemendy even accused Tarski of committing what he called "Tarski's fallacy", inferring from the necessary truth of a conditional the necessary truth of its consequent given the truth of its antecedent. Specifically, Tarski claims (2002/1936, pp. 186–187) that, if a sentence follows logically in his sense from true sentences, then it must be true. Rephrased in contemporary terms, the claim would be that a sentence that is true on every interpretation on which one or more sentences are true must be true on any interpretation on which the latter sentences are true. Or, to put it in the form of an argument:

> Sentence X is true in every interpretation in which the sentences of class $\mathcal{K}$ are true.
>
> Therefore, if the sentences of class $\mathcal{K}$ are true in an interpretation, then the sentence X must be true in that interpretation.

It is not obvious that this argument is valid, since its premiss is assertoric and its conclusion is apodictic. What licenses the transition from a claim about how things are as a matter of fact to how things must be? Defenders of Tarski's claim, such as Gila Sher (1996), have argued that, because all the extra-logical constants in the sentences are subject to reinterpretation, and variation of the domain is possible, the absence of a counter-interpretation is not just a matter of empirical fact, but a matter of logical necessity. Sher's argument for this claim depends on an appeal to set theory, which thus becomes in a certain sense prior to logic.

It is thus possible to defend the claim, ubiquitous in contemporary work in logic, that absence of a counter-interpretation is a matter of necessity and not just a matter of fact.

What about a similar claim for extensions of the model-theoretic conception to non-logical consequence?

4. Revision and expansion of substitutional, formal and model-theoretic conceptions of consequence

In previous work (Hitchcock 1998), I proposed a revision of the existing generic conception of logical consequence and an extension of the revised generic conception to cover what I there called, following George (1972), 'enthymematic consequence'. In the present article I shall review and then modify that proposal, in the process answering some questions left open in its concluding section.

I begin by distinguishing five specific conceptions of logical consequence that one can find in the literature.[5]

1. According to the *deducibility conception*, a sentence is a logical consequence of one or more sentences if and only if it can be deduced from them in a formal system. The deducibility conception is usually taken to be parasitic on the model-theoretic conception, in the sense that the soundness of a formal system is proved by treating the model-theoretic criterion of logical consequence as the "gold standard" and showing that any sentence deducible from given sentences using the rules of the formal system does follow from them in the model-theoretic sense: if the sentence is deducible, then there is no interpretation on which it is untrue when the given sentences are true. Likewise, the completeness of a formal system is shown by proving that any sentence of the formal language deducible from one or more sentences is true in any interpretation that makes true the sentence or sentences from which it is deduced. The deducibility conception can however be taken as basic if one takes the meaning of a sentence to be what it implies, as proposed by Gentzen (1969/1935) in his formulation of natural deduction systems and sequent calculi with a pair of rules for each logical constant, a so-called "elimination rule" indicating what one may deduce from a sentence in which that constant is the main logical operator and a correlative "introduction rule" indicating what one may deduce such

[5] The description of these five conceptions and my remarks about them incorporate material from pages 20–24 of (Hitchcock, 1998).

a sentence from. Gentzen's proposal has been elaborated and extended from logical constants to all terms by Wilfrid Sellars (1953) and Robert Brandom (1994, 2000) in what Brandom calls "inferential semantics". We will return to the Sellars-Brandom proposal later.

2. According to the *modal conception*, articulated for example by Stephen Read (1994), an argument's conclusion follows logically from its premisses if and only if there is no possible situation where the premisses are true and the conclusion untrue. The modal conception is identical to the conception of consequence as necessary truth-preservation identified at the beginning of the present article. Proponents of this conception are distinguished from proponents of the other four conceptions now being distinguished in their willingness to apply the conception directly rather than giving an account of it in terms of deducibility or some other relation. The modal conception can account for cases where a conclusion follows necessarily from given premisses, even though it does not follow formally. That is, it is not deducible from them in a formal system, nor does it follow if substitution or (re-)interpretation is allowed on all extra-logical constants. Thus the conclusion of the argument 'Iain is a bachelor, so Iain is unmarried' follows from its premiss, because the meanings of the terms 'bachelor' and 'unmarried' rule out any situation in which the premiss is true and the conclusion untrue. Read (1994, p. 257) explicitly argues against the claim that the conclusion of this argument only really follows when a 'suppressed premiss' that all bachelors are unmarried is made explicit. The modal conception requires clarification of what sense of 'possible' is involved. Its proponents seem to intend a sense which is relative to the meaning of an argument's component sentences. So their conception might be reworded more precisely as the notion that an argument's conclusion follows from its premiss(es) if their meaning is incompatible with there being a situation where the premisses are true and the conclusion untrue. If so, the modal conception coincides in its extension with the Sellars-Brandom proposal for an inferential semantics. However, it is possible to embrace the modal conception without making the deducibility relationships of a sentence semantically prior to its truth-conditions.

3. On the *substitutional conception*, a conclusion is a logical consequence of given premisses if and only if there is no substitution on its extra-logical constants which produces an argument with true premisses and an untrue conclusion. This conception stems ultimately from Bolzano (1972/1837), who according to George's reconstruction (1972, 1983) accommodated not only logical consequence but also enthymematic consequence, by allowing substitution on some but not all extra-logical constants. Bolzano's version

of the substitutional conception is immune to Tarski's objection that a language might lack names for objects that would constitute a counter-example, because Bolzano postulated a realm of abstract ideas on which substitutions were to be made. However, Quine has argued that the substitutional conception of logical truth is equivalent to the model-theoretic conception, provided that the language used for substitution is rich enough for elementary number theory (Quine 1970, pp. 53–55). Presumably his argument would apply as well to a comparison between the substitutional and the model-theoretic conceptions of logical consequence.

4. On the *formal conception*, a conclusion follows logically from given premisses if and only if the argument is an instance of a form of argument which has no instances with true premisses and an untrue conclusion. A form of argument is a linguistic schema which includes at least one meta-linguistic variable but no extra-logical constants and from which an argument can be derived by replacing all occurrences of each variable with the same extra-logical constant or grammatically parallel complex content expression. The formal conception is open to the same objection from the possible poverty of a language as the substitutional conception, and can make use of the same reply.

5. On the *model-theoretic conception*, a sentence X follows logically from given sentences if and only if every true interpretation of those sentences is also a true interpretation of the sentence X. As pointed out earlier, this conception is standard in contemporary work in formal logic.

All five conceptions give rise to two paradoxes, which are in fact generic problems with the conception of the consequence relation as one in which it is impossible for the premisses to be true and the conclusion untrue. If the word 'and' in this standard conception is construed as expressing truth-functional conjunction, then this conception implies that any conclusion at all follows from premisses which cannot all be true: if it is impossible for the premisses to be true, then it is *a fortiori* impossible for the premisses to be true and the conclusion untrue. Thus the sentence 'Amsterdam is the capital of Canada' would follow from the sentences 'There are living organisms beyond the planet earth' and 'There are no living organisms beyond the planet earth'. Intuitively, however, it does not follow, since the sentences about extra-terrestrial life have nothing to do with whether Amsterdam is the capital of Canada. The medieval rule of inference *ex falso quodlibet* ('from a falsehood anything follows') should be rejected. Similarly, the standard generic conception implies that any conclusion which must be true (i.e. cannot be untrue) follows from any premisses whatever: if it is impossible for the conclusion to be untrue, then it is *a fortiori* im-

possible for the premisses to be true and the conclusion untrue. Thus the sentence 'whenever it is raining, it is raining' would follow from the sentence 'The Hague and Amsterdam are capitals of the Netherlands'. Intuitively, however, it does not follw, since the sentence about the capitals of the Netherlands has nothing to do with the weather. The medieval rule of inference *ex quolibet verum* ('from anything a truth follows') should likewise be rejected.[6]

One could avoid these paradoxes by adding two requirements for consequence, that it is possible that all the premisses are true and that it is possible that the conclusion be untrue. This strategy, however, would introduce new paradoxes, by barring a set of sentences that cannot all be true from having any consequences and barring any sentence that must be true from being a consequence of any set of sentences. Intuitively, impossibilities do have consequences, and necessities can be consequences. For example, any sentence is a consequence of itself, even if it cannot be true or must be true. To avoid the new paradoxes, one needs to introduce the concept of a content expression, which I first used in (Hitchcock 1985). A content expression is an expression in a sentence that can be replaced by an extra-logical constant without loss of grammaticality. Content expressions may themselves be extra-logical constants, in which case they are atomic content expressions. Otherwise they are molecular content expressions. A whole sentence is a content expression, assuming that one's language possesses sentence constants. A conjunctive predicate like 'square and circular' is a content expression. And so forth. We also need the concept of a set of content expressions that exhausts the extra-logical constants in a set of sentences, in the sense that replacement of these content expressions in the sentences in question with metalinguistic variables produces a set of sentence schemata in which there is no extra-logical constant (cf. Hitchcock 1998, pp. 25–26).

With the concept of a content expression, we can redefine the substitutional, formal and model-theoretic conceptions of logical consequence so as to avoid both pairs of paradoxes.

3′. On the *revised substitutional conception*, a sentence X is a logical consequence of the sentences of the class $\mathcal{K}$ if and only if there is an exhaustive set of content expressions in these sentences on which no uniform substitution produces an untrue sentence X' and a class $\mathcal{K}'$ of true sentences, at least one such substitution produces a class $\mathcal{K}''$ of true sentences, and at least one such substitution produces an untrue sentence X''.

[6] The preceding paragraph summarizes and adapts (Hitchcock 1998, pp. 24–25).

4′. On the *revised formal conception*, a sentence X is a logical consequence of the sentences of the class $\mathcal{K}$ if and only if they are instances of a set of sentence schemata in which there are no extra-logical constants and for which no instance consists of an untrue sentence X' and a class $\mathcal{K}'$ of true sentences, at least one instance includes a class $\mathcal{K}''$ of true sentences, and at least one instance includes an untrue sentence X'''. This conception is essentially that advanced by Smiley (1959, p. 240).

5′. On the *revised model-theoretic conception*, sentence X is a logical consequence of the sentences of the class $\mathcal{K}$ if and only if there is a an exhaustive set of content expressions in these sentences for which no interpretation produces an untrue sentence X' and a class $\mathcal{K}'$ of true sentences, at least one interpretation produces a class $\mathcal{K}''$ of true sentences, and at least one interpretation produces an untrue sentence X'''. The concept of an interpretation can be redefined so that interpretations assign objects to content expressions as wholes, or alternatively one can allow replacement of molecular content expressions in the set by extra-logical constants of the same grammatical type and apply the model-theoretic definition to the sentences thus constructed.[7]

All three conceptions imply a relevance condition of topical overlap between implying sentences and implied sentence. That is, if the implied sentence X contains an extra-logical constant, there is at least one extra-logical constant that occurs both in the sentence X and in at least one sentence of the class $\mathcal{K}$.[8]

Development of these revised conceptions of logical consequence permits their natural extension to non-logical consequence, simply by dropping the requirement in each definition that the set of content expressions be exhaustive. Logical consequence would then be just the special case in which the set with reference to which the definition is met subjects all the extra-logical constants in the sentences, either directly or by their inclusion in a molecular content expression in the set, to substitution or replacement by another instance or (re-)interpretation. It should be noted that the additional clauses in the definitions, added to rule out the paradoxes of *ex falso quodlibet* and

[7] These revised conceptions adapt the conceptions found in Hitchcock (1998, p. 26), with the additional constraint that the set of content expressions is exhaustive.

[8] For a proof with respect to Bolzano's substitutional conception, applied to the language of classical propositional logic, see George (1983). The qualification that the implied sentence contains an extra-logical constant is needed to accommodate cases where the consequence relation obtains but the implied sentence contains no extra-logical constants. For example, the sentence 'there is at least one object' follows from the sentence 'there are at least two objects' on any of the three revised conceptions, even though it contains no extra-logical constants.

ex quolibet verum, automatically rule out so-called 'material implication' (the medievals' *consequentia materialis*) as a consequence relation. For, if there are extra-logical constants in the sentence X or the sentences of the class $\mathcal{K}$, then the mere fact that we do not have the sentences of $\mathcal{K}$ true and sentence X untrue is not sufficient for consequence on any of the revised conceptions. For, if the set of content expressions with respect to which the criterion for consequence is to be applied is empty, then either the clause requiring X to have an untrue parallel will fail or the class requiring the sentences of the class K to have parallel sentences that are all true will fail. That is, the only parallel in this case for X is X itself and the only parallel sentences of the sentences of the class $\mathcal{K}$ are those sentences themselves. But, by hypothesis, either X is true or not all the sentences of the class $\mathcal{K}$ are true, or both.

The revised conceptions of consequence thus make it easier to test for non-logical consequence. It is necessary to consider only sets of content expressions that include at least one expression common to a premiss and the conclusion of an argument. As a matter of heuristics, the best strategy to use in seeking consequence-implying content expressions in an argument is to generalize as broadly as possible with respect to all the maximal repeated content expressions, whether these are repeated within the premisses or between a premiss and a conclusion. If the conclusion turns out not to follow with respect to this set, one can then try narrower generalizations or smaller sets of content expressions or less maximal content expressions, always retaining at least one content expression common to a premiss and a conclusion (Hitchcock 1985, 1998).

5. The problem of contingent non-trivial truth-preservation

With this revised and expanded conception of consequence in place, we can return to the question posed earlier: Is consequence a mere matter of fact or also a matter of necessity?

The answer is in fact quite obvious. Not only on the revised and expanded model-theoretic conception just articulated, but also on the parallel substitutional and formal conceptions, there are cases where a sentence X is a consequence of the sentences of some class $\mathcal{K}$ as a mere matter of contingent fact and not as a matter of necessity. For example, no president of the United States of America in the first 230 years of its existence was a woman. This fact is contingent, but it nevertheless underwrites a consequence relation between the sentence 'Abraham Lincoln was president of the United

States of America for a period during the first 230 years of its existence' and the sentence 'Abraham Lincoln was not a woman'. For, given the contingent fact, no substitution on the name 'Abraham Lincoln' will produce parallel sentences with the first untrue and the second true; furthermore, the substitution of 'Hubert Humphrey' for 'Abraham Lincoln' produces an untrue parallel to the first sentence and the second sentence is already true. Similarly for the sentence schemata 'x was president of the United States of America for a period during the first 230 years of its existence' and 'x was not a woman', and for (re-)interpretations of the name 'Abraham Lincoln'.

Does the contingency of the revised and expanded conception of consequence matter? After all, a contingent fact gives just as strong an assurance of truth-preservation as a necessary connection. Assurance is weakened only if there is some doubt about the truth of the inference-underwriting sentence, but doubt is possible with respect to necessary truths as well as with respect to contingent ones.

Additional support for a consequence relation that can obtain merely contingently comes from the strikingly close match between the covering generalization that underwrites each such consequence and the supposed 'unstated premiss' that skilled argument analysts intuitively supply. For example, application of the revised and expanded conception of consequence to arguments traditionally regarded as incomplete Aristotelian syllogisms will generate a covering generalization, with respect to the term shared between premiss and conclusion, that is logically equivalent in all cases to a sentence whose addition as a premiss would transform it into a complete Aristotelian syllogism. As another example, the revised and expanded conception of consequence was easily applied to all but one of a sample of 50 arguments in scholarly books selected by random methods, as well as to all of a sample of 37 arguments uttered in phone calls to radio and television talk shows, also selected by random methods (Hitchcock 2002, forthcoming).[9]

Furthermore, reinterpretation of a supposed unstated premiss as a claim underwriting a consequence relation explains why the supposed unstated premiss is typically a covering generalization of the stated argument, or something from which such a covering generalization can be derived, rather than the "logical minimum" (Van Eemeren and Grootendorst 1992,

[9] In testing the applicability of my conception of good inference to actual arguments that scholars and callers to talk shows advance, I used an even more expanded conception that allowed for probabilistic and presumptive inferences, underwritten respectively by for-the-most-part and *ceteris paribus* covering generalizations. In the present article, I do not discuss this further expansion of the concept of consequence.

pp. 64–67) whose addition as a premiss would make the stated argument formally valid. That logical minimum is the "associated (material) conditional" (Hitchcock 1985) of the argument, the ungeneralized negation of the conjunction of the conjunction of the premisses and the negation of the conclusion. Someone who reasons to a conclusion or adduces evidence as conclusively supporting a claim does more than rule out the combination of true reasons (evidence) and untrue conclusion (claim). Such a person makes a commitment to the same sort of inference in parallel cases, as is shown by the strategy of "refutation by logical analogy", constructing a parallel argument with true premisses and a false conclusion. Thus the person is implicitly using a general rule of inference, which is typically not purely formal. If Mary's mother tells her, "You can't have dessert, because you didn't eat your peas", Mary can quite legitimately reply: "But Johnny got dessert, and he didn't eat his peas." It would be "illogical" for the mother to reply, "I'm talking about you, not about Johnny". She has committed herself to the form of argument, 'x can't have dessert, because x did not eat x's peas', and she must explain why this form of argument does not apply to Johnny when it applies to Mary.

It turns out, however, that acceptance of merely contingent consequence relations has counter-intuitive implications in particular cases. On any of the revised and expanded conceptions of consequence, the sentence 'Napoleon was imprisoned on Elba' follows from the sentences 'Napoleon ruled France' and 'Napoleon was born in Corsica'. For, since Napoleon has been (I am assuming) the only Corsican-born ruler of France, and he was in fact imprisoned on Elba, and many other people have not been imprisoned on Elba, there is no re-interpretation of the name 'Napoleon' on which 'Napoleon was imprisoned on Elba' is untrue but 'Napoleon ruled France' and 'Napoleon was born in Corsica' are true, even though there is a re-interpretation of 'Napoleon' on which 'Napoleon was imprisoned on Elba' is untrue and there is a re-interpretation of 'Napoleon' (namely, the trivial "re-interpretation" on which 'Napoleon' refers to Napoleon) on which 'Napoleon ruled France' and 'Napoleon was born in Corsica' are true. But intuitively, 'Napoleon was imprisoned on Elba' does not follow from the sentences 'Napoleon ruled France' and 'Napoleon was born in Corsica'. The mere fact that Napoleon was born in Corsica and ruled France, we might say, does not count as evidence that he was imprisoned in Elba, does not entitle us to conclude that he was imprisoned in Elba.[10]

[10] The reflections in the preceding paragraph were stimulated by an article by Robert Pinto (2006) and by subsequent correspondence with Pinto and James B. Freeman.

An initial response to this difficulty might be to move back from the truth-based conception of consequence to the concept of necessity that it was trying to explicate. Such a strategy would force us to abandon the substitutional and model-theoretic versions of the revised and expanded conception of consequence, and to focus on the formal version. For substitutions and (re-)interpretations shed no new light on whether the clauses of the definition hold necessarily or merely contingently. With the formal version, however, we can ask whether the non-existence of an instance with untrue X and true sentences of a class $\mathcal{K}$ is a matter of necessity, by asking counter-factually whether there could be such an instance, even if as a matter of fact there is none. That is, we would be testing whether the covering generalization is lawlike rather than accidental, in a way that would support counter-factual inferences. We can see immediately that our two examples with contingently true generalizations would fail this test. If Walter Mondale had been elected president in 1984 and had died in office, with the result that his running mate Geraldine Ferraro became president of the United States of America, it would not be true that Geraldine Ferraro was not a woman. Similarly, we could tell a variant story of the history of France in which it had a ruler who was born in Corsica but was never imprisoned on Elba; indeed, if by chance some other ruler of France than Napoleon was born in Corsica, it is most unlikely that he would have been imprisoned on Elba.

This strategy takes us back to the difficulty of deciding when non-trivial truth-preservation is a matter of necessity. Sellars (1953) and Brandom (1994, 2000) propose to construe all such necessity as a matter of meaning, and in Brandom's case to get rid of "representational semantics" based on the concept of truth in favour of "inferential semantics" based on the concept of necessary inference. This approach accommodates our practices of reasoning and arguing much better than a formal or logical conception of consequence. But it does so at a cost. First, consequence relations that are most naturally understood as grounded in some physical necessity (such as an objects' exercise of gravitational attraction being a consequence of its mass) or legal necessity (such as a person's being at least 35 years of age being a consequence of the person's having been elected president of the United States of America) are implausibly treated as grounded in the meanings of the related sentences.[11] Second, having discarded representational seman-

[11] In the preceding sentence, I use the word 'consequence' in an inferential rather than a causal sense. To be a consequence of something in the inferential sense is to be legitimately inferable from it.

tics, Brandom is left with nothing to ground our inferential practices except our inferential practices. This strategy flies in the face of our ordinary way of justifying our inferences. If I argue that John F. Kennedy must have been at least 35 years old by the end of 1960, since he was elected U.S. president in November 1960, and you ask me how that follows, I will most naturally point to the provision in section 1 of Article II of the U.S. constitution that "neither shall any person be eligible to that office [of president – D. H.] who shall not have attained to the age of thirty-five years". It is the fact of this constitutional requirement that grounds the inferential practice that I exemplify in this situation. It would be quixotic to treat the clause in the constitution as a product of our inferential practices.

If we hold on to a representational semantics and treat our inferential practices as grounded in that semantics, then we can rule out merely contingent consequence relations by requiring that the schema in virtue of which X is a consequence of the sentences of some class $\mathcal{K}$ have no counter-instances not only as a matter of fact but also necessarily. We can leave open-ended the types of necessarily true generalizations that can underwrite a consequence relation, except that we exclude deontic necessities. Any type of necessity that implies actuality will do. Thus the necessity of a true covering generalization that underwrites a consequence relation may be logical, semantic, physical, mathematical, biological, constitutional, and so forth.

By requiring such a true covering generalization to be lawlike, supporting counter-factual instances, have we given up too much? Counter-examples in the opposite direction, where the only true covering generalizations are merely contingent but a consequence relation seems to obtain, come to mind. The sentence "Jesus was mortal" seems intuitively to follow from the sentence "All humans are mortal". But the minimal non-trivially true covering generalization for an argument from "All human are mortal" to "Jesus is mortal" is the generalization "If all humans are F, then Jesus is F", which is logically equivalent to the sentence "Jesus is human".. And the sentence "Jesus is human" is arguably contingent. Some Christian theologians may take it to be false, supposing that the divinity of Jesus is incompatible with his (full) humanity. Or perhaps Jesus was an alien, and lacked at least one property shared by all human beings.

If such counter-examples are persuasive, they raise the challenge of discovering a principled intermediate position between a very broad consequence relation groundable in merely contingent true covering generalizations and a somewhat narrower consequence relation that requires an inference-licensing covering generalization to be true as a matter of necessity.

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