

WITTGENSTEIN'S TRACTARIAN APPRENTICESHIP

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The years since the publication of Wittgenstein's *Tractatus* have produced a good many interpretations of its central tenets. Time has produced something of a consensus concerning the nature of the Tractarian criticisms of Russell's philosophy. Recent work on Russell's philosophy of logic reveals, however, that the agreed account of Tractarian criticisms relies upon characterizing Russell with positions he did not hold.

Wittgenstein's *Tractatus*¹ is an extremely condensed and aphoristic work. The polemic is nowhere explicit. Wittgenstein published little in his life, but left voluminous work-notes of oracular pronouncements, comments, and aphorisms. Consequently, the years since the publication of the *Tractatus* have produced a good many interpretations of its central tenets. Time has produced something of a consensus, however, concerning the nature of the Tractarian criticisms of Russell's philosophy of logic and mathematics, and this is the subject of the present paper. Russell also left voluminous work-sheets, and these have shed an entirely new light on the philosophical evolution of Whitehead and Russell's 1910 *Principia Mathematica*. Much of the literature on the relationship between Russell's philosophy and Wittgenstein's *Tracta-*

¹ Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trans. D. F. Pears and B. F. McGuinness (New York, Routledge & Kegan Paul, 1969). Hereafter referred to as *TLP*.

tus is concerned to distinguish Tractarian doctrines from the phenomenological epistemology of Russell's 1914 *Our Knowledge of the External World* and a reading of his 1918 *Logical Atomism* lectures that portrays Russell's atomism as a part of this sort of epistemological programme. But the proper context for comparison is the technical and philosophical positions leading up to and espoused in *Principia Mathematica*. Put starkly, recent work on Russell's philosophy of logic reveals that the agreed account of Tractarian criticisms relies upon characterizing Russell with positions he did not hold.

The Master Motif

As is well known, the *Tractatus* was first published in 1921 with an introduction by Russell. When it was translated into German, Wittgenstein wrote that all the refinement of Russell's English style was lost and what remained was superficiality and misunderstanding.² With Wittgenstein's denouncement of Russell's introduction, it has come to be believed that the Tractarian "fundamental idea" that the "logical constants" are not representatives (TLP 4.0312) should be viewed as antithetical to Russell's views. Pears writes:

A short answer to the question about the relation between Wittgenstein's view of logic in the *Tractatus* and Russell's ... is that they were really opposed to one another. Russell believed that the logician's task is to carry out a survey of "logical objects", some of which are forms while others are the real counterparts of logical connectives. Wittgenstein's view was that there are no logical objects ... and logical connectives do not stand for anything in the world.³

Though there is some truth in Pears's short answer, it is misleading in the extreme to characterize Russell as holding that the logical connectives stand for logical objects. In his book, *Wittgenstein's Place in Twentieth Century Philosophy*, Hacker puts it as follows:

Both Frege and Russell conceived of the logical connectives as names of logical entities.... Russell construed them as naming functions from propositions to propositions. This conception was linked to their idea that propositions are

² Letter to Russell of 6 May 1920, in L. Wittgenstein, *Cambridge Letters*, ed. Brian McGuinness and G. H. von Wright (Oxford: Blackwell, 1995), p. 154.

³ David Pears, *The False Prison*, Vol. 1 (Oxford: Clarendon P., 1987), p. 25.

names of truth-values (Frege) or complexes (Russell). But it is a dire error to think that “ $p \vee q$ ” has the same logical form as ‘ aRb ’.⁴

The dire error here is Hacker's. In *Principia*, Russell's former ontology of propositions had been *explicitly* abandoned, and the work adopted the wedge (“ \vee ”) and the tilde (“ \sim ”) as *statement* connectives in just the modern sense. Hacker awards Wittgenstein with the “achievement” of having “... freed himself of many of Russell's deep confusions about the role of logical expressions ...” (p. 22). And yet Russell was not guilty of those confusions. Unfortunately, such contortions of the historical Russell have carried the day.

Ray Monk's recent biographies nicely illustrate just how garbled the history has become. Wittgenstein began as Russell's student in October of 1911. On Monk's account, he pursued his studies in mathematical logic with such vigour that, by the end of one term of study, Russell was to say that he had learnt all he had to teach, and indeed gone further.⁵ Monk would have us believe that Wittgenstein learned everything Russell knew about mathematical logic in less than one year. By January 1913, Monk proclaims that cooperation between the two had come to an end. “In the field of logic, Wittgenstein, far from being Russell's student, had become Russell's teacher” (*ibid.*).

The case that Wittgenstein became Russell's teacher is invariably built around Russell's letters to Ottoline Morrell. Much has been made, for instance, of Wittgenstein's criticisms of Russell's multiple-relation theory of judgment, a theory first espoused in *Principia* and later worked out in Russell's 1913 manuscript for a book on the theory of knowledge. Russell abandoned his project in the wake of a storm of protest from Wittgenstein. One can find Russell writing Ottoline that Wittgenstein's criticisms “... were an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy” (*Auto.*, 1: 57). When Russell writes Ottoline of despair of ever again doing fundamental work in philosophy, of suicidal depression over failed work—

⁴ P. M. S. Hacker, *Wittgenstein's Place in Twentieth-Century Analytic Philosophy* (Oxford: Blackwell, 1996), p. 28.

⁵ Ray Monk, *Ludwig Wittgenstein: the Duty of Genius* (New York: Free P., 1990), p. 72.

feelings which, he says, were caused by exasperating exchanges with the ill-tempered Wittgenstein—Monk seizes upon this as evidence of Wittgenstein’s philosophical and *intellectual* superiority.

In truth, there has to be an alternative explanation of the letters to Ottoline, for Russell’s unpublished manuscripts reveal that Monk’s account of Wittgenstein’s objection to Russell’s multiple-relation theory is mistaken. Wittgenstein’s objection was supposedly that to rule out non-sensical belief Russell must rely on *Principia*’s theory of types—a theory Wittgenstein rejected: “... all theory of types”, he wrote to Russell, “must be done away with by a theory of symbolism”.⁶ Monk spins this as follows:

In the face of such a sweeping dismissal of his theory, Russell might have been expected to present a spirited defence of his position—or at least some tough questions as to how his logicist foundations of mathematics might avoid contradiction *without* a theory of types. But he had by this time abandoned logic almost entirely. (Monk, 1: 71)

How ironical this is. Russell’s unpublished manuscripts show that the idea that grammar must do justice to type distinctions, since there cannot be different types of entities, is a position Wittgenstein *inherited* from Russell. Assuming that Wittgenstein understood Russell, his objection to the multiple-relation theory could not have been to *Principia*’s order/type distinctions of “propositional functions” because they *are* built into grammar (i.e., the significance conditions of the use of predicate variables). In truth, Wittgenstein’s objection was that the predicable nature of a universal, as opposed to a particular, is *another* “type” distinction that must *also* be built into logical grammar. It is not at all odd or perplexing, then, that Monk cannot find Russell bridleing at Wittgenstein’s alleged “sweeping dismissal” of type-theory. There simply was no such dismissal. Monk’s biographies conflate Russell’s personal and emotional turmoil over failed relationships with Ottoline and his wife Alys, with the intellectual issues pertaining to his philosophy. Monk relies upon an account of Russell’s philosophy, which, albeit entrenched in the secondary literature, is rapidly changing in light of Russell’s unpublished

⁶ Wittgenstein, *Notebooks 1914–1916*, ed. G. H. von Wright and G. E. M. Anscombe (Chicago: U. of Chicago P., 1979), p. 105.

manuscripts.

How then shall we explain Russell's admissions to Ottoline? Russell's emotional life was a shambles during this period. Lady Ottoline was his angel of mercy, his hope in new love for release from despair and suffering. Yet Ottoline came to be aloof to Russell's pouring sentimentality and took many of his letters as attempts to cajole her sympathy in hopes of making her commit to a life with him. Russell was then 41 and wanted a companion and children.⁷ Ottoline, a woman of no small experience with men, had stroked not a few egos claiming despair over what they regarded as failed work. She had no intention of abandoning her life with Philip Morrell and her daughter, and soon tired of Russell's plotting to drive her away from them and his maudlin declarations of suicidal feelings.

Monk relishes Russell's remark to Ottoline that "Wittgenstein has persuaded me that the early parts of *Principia Mathematica* are very inexact, but fortunately it is his business to put them right, not mine",⁸ as if it provides evidence of Russell's realizing his inferiority to Wittgenstein in mathematical logic and his bequeathing the foundations of logic and mathematics to his pupil.⁹ Monk writes:

These remarks are revealing. They show how Russell was still inclined to look upon Wittgenstein's work as a kind of "fine tuning" of his own. He talks as if the inexactitude of the early parts of *Principia* is a mere detail, but those early parts contain the very foundation upon which the whole of the rest was built. And Wittgenstein was not repairing it, as Russell continued to think, he was demolishing it altogether.

To be sure, Pinsent's diary for August 1913 records Wittgenstein being charged with the task of "rewriting the first eleven chapters of *Principia's* vol. 1".¹⁰ But Monk's interpretation distorts the historical situation greatly. Comments like his have become part of the folklore of Wittgenstein, but they cannot withstand scrutiny.

Sheffer read a paper to the American Mathematical Society on 31 December 1912 showing that *Principia's* truth-functions can be expressed

⁷ Russell to Ottoline Morrell, 18 August 1913; *SLBR*, 1: 469.

⁸ Letter to Morrell, 23 Feb. 1913; *SLBR*, 1: 448.

⁹ See, e.g., Moorehead, p. 174; Monk, *Wittgenstein*, p. 72; Monk, 1: 290.

¹⁰ See Brian McGuinness, *Wittgenstein: a Life* (London: Duckworth, 1988), p. 180.

via just one logical connective.¹¹ Having received Sheffer's paper on 15 April 1913, Russell, it is certain, would have shared it with Wittgenstein. Indeed, there is evidence. Russell read a paper entitled "On Matter" to the Cambridge Moral Sciences Club in April of 1912, and Wittgenstein attended. On the back page of a version of Russell's paper entitled "On Matter—The Problem Stated", there are jottings in both Russell's and Wittgenstein's hands suggesting a knowledge of Sheffer's work.¹² It would not in the least belittle Russell's stature as mentor if he discussed whether Wittgenstein might make it part of his dissertation (which was required of a Research Student)¹³ to work out a new reduction of *Principia's* *1–5 based on Sheffer's single connective. In fact, Wittgenstein's famous self-imposed isolation in Norway "until he has solved *all* the problems of logic"¹⁴ failed to produce the reduction that Nicod found in 1916.¹⁵ In the 1925 introduction to the second edition of *Principia*, Russell even recommends that Sheffer "rewrite" the first parts of *Principia* using the stroke (*PM*, I: xv).

There is no question that Wittgenstein had ideas for improving the philosophical foundation of *Principia* and that Russell was enticed by them. For example, Wittgenstein proclaimed that a proper elimination of the identity symbol would obviate *Principia's* need to add statements of the infinity of individuals as antecedents of central theorems concerning inductive cardinals. He also advocated an extensionalist position that "a function can appear only through its values" and suggested that this would avoid the need for *Principia's* Reducibility Principle. But observe that in *Principia* itself, Russell maintained that some formulation embodying type structures must be correct, and expressed a hope that with further work in the area, the axiom of reducibility might yet be sup-

¹¹ H. M. Sheffer, "A Set of Five Independent Postulates for Boolean Algebras, with Application to Logical Constants", *Transactions of the American Mathematical Society*, 14 (1913): 481–8.

¹² See McGuinness, *Wittgenstein: a Life*, p. 160.

¹³ Moore, in fact, seems to have viewed Wittgenstein's early notes on logic in this way. See McGuinness, *Wittgenstein: a Life*, p. 199.

¹⁴ Russell to Lucy Martin Donnelly, 19 Oct. 1913; quoted in McGuinness, *Wittgenstein: a Life*, p. 184.

¹⁵ Jean Nicod, "A Reduction in the Number of the Primitive Propositions of Logic", *Proceedings of the Cambridge Philosophical Society*, 19 (1917): 32–41. The paper was read before the Society on 30 October 1916.

planted. He acknowledged that there remained philosophical difficulties with *Principia's* reliance on reducibility and invited *all* his readers to work on them (*PM*, 1: vii, xiv, 60). The fact that Russell was interested in Wittgenstein's ideas for solving philosophical problems remaining in the *Principia* provides no basis for concluding that Russell had become his "pupil".

Research on Russell's voluminous work-notes has shed a flood of new light on his views. Without carefully considering this evidence, one cannot be in a position to assess Wittgenstein's points of agreement or disagreement with Russell, much less to proclaim with Monk that Wittgenstein was Russell's intellectual "master", or with Hacker that the *Tractatus* was an "achievement" in the philosophy of logic.

Russell's World as Wittgenstein Found It

A new picture of the intellectual and philosophical relationship between Russell and Wittgenstein is needed. The picture I shall sketch below is based upon the new analysis of the historical evolution of *Principia Mathematica* set out in my book, *Russell's Hidden Substitutional Theory*.¹⁶

For a great many years, *Principia* has been viewed as advocating ontological reductions—classes are to be identified with certain propositional functions (attributes in intension), propositions are identified with judgments, natural cardinal numbers are reduced to classes, ordinal numbers are reduced to (identified with) classes of well-ordering relations-in-extension, and so on. Similarly, physical continuants are reduced to bundles of actual and possible sense-data (in Russell's 1914 *Our Knowledge of the External World*). This reading of *Principia* undoubtedly stems from attempts to recover the philosophy of *Principia* by understanding it in the context of British empiricism and Russell's 1917–18 logical atomism. Russell described the method thus:

Given a set of propositions nominally dealing with the supposed inferred entities, we observe the properties which are required of the supposed entities in order to make these propositions true. By dint of a little logical ingenuity, we then construct some logical function of less hypothetical entities which has the requisite properties. This constructed function we substitute for the supposed

¹⁶ Landini, *Russell's Hidden Substitutional Theory* (New York: Oxford U. P., 1998).

inferred entities, and thereby obtain a new and less doubtful interpretation of the body of propositions in question.¹⁷

In his 1924 “Logical Atomism”, Russell puts it thus:

One very important heuristic maxim which Dr. Whitehead and I found, by experience, to be applicable in mathematical logic, and have since applied in various other fields, is a form of Ockham’s razor. When some set of supposed entities has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical structures without altering any of the detail of the body of propositions in question. This is an economy, because the entities with neat logical properties are always inferred, and if the propositions in which they occur can be interpreted without making this inference, the ground for the inference fails, and our body of propositions is secured against the need for a doubtful step. The principle may be stated in the form: “Wherever possible, substitute constructions out of known entities for inferences to unknown entities.” (*LK*, p. 326; *Papers* 9: 164)

Russell’s method of logical construction has been taken to be that inference to (or postulation of) new entities is to be replaced by ontological reductions to entities such as sense-data, and to propositional functions which are known by acquaintance.

Indeed, Pears makes an epistemological principle of acquaintance with sense-data part of the very definition of Russell’s logical atomism. He writes:

“Logical atomism” is Russell’s name for the theory that there is a limit to the analysis of factual language, a limit at which all sentences will consist of words designating simple things.... His theory of knowledge led him to claim that the only simple particulars that we know are sense-data, and that the only simple qualities and relations we know are certain qualities and relations of sense-data. Their simple qualities and relations are those with which we have to achieve acquaintance in order to understand the words designating them. This fixes the character of his logical atomism. It is a version of empiricism and it uses a criterion of simplicity based on the exigencies of learning meanings.... The doctrine of forced acquaintance is the foundation of Russell’s logical atomism. (*The False Prison*, 1: 63)

¹⁷ Russell, “On the Relation of Sense-Data to Physics”, *ML*, p. 156; *Papers* 8: 12.

As Pears sees it, Russell operates with a criterion of simplicity that allows him to identify logical atoms as things with which we are familiar, namely sense-data and their properties (*ibid.*, 1: 68). The foundation of Russell's atomism, on this view, is an empiricist principle of acquaintance; it takes sense-data as its logical atoms, and offers a foundational and empiricistic construction of factual language. In contrast, Pears characterizes Wittgenstein's atomism quite differently: Wittgenstein was unconcerned with epistemology, and he endeavoured to set the "limits of thought" and the logical scaffolding of thought, language, and the world.

This popular characterization of Russell's logical atomism and of the differences between Russell and Wittgenstein is quite mistaken. The existence of Russell's largely unpublished substitutional theory has revealed new evidence concerning the historical evolution of *Principia Mathematica*. To see this we must understand the historical evolution of *Principia* from Russell's substitutional theory—a theory which cannot be interpreted as an ontological reduction but must be understood as an ontological elimination. The interpretation of *Principia* as offering an ontological reduction is mistaken.

It is not well known that prior to *Principia* Russell had genuinely solved the paradoxes of classes and attributes by means of his "substitutional theory" of propositions.¹⁸ It is not a "theory" of types of objects, but rather a dissolution of the paradoxes by means of an eliminativistic ontological analysis and reconstruction. The source of Russell's discovery of the possibility of such an eliminativistic reconstruction was his Doctrine of the Unrestricted Variable. Russell's adherence to the doctrine was the guiding principle that led him to generate constructions

¹⁸ *Russell's Hidden Substitutional Theory*. The substitutional theory was plagued by a paradox of general propositions that I call the " p_o/a_o paradox". In his 1906 paper "On 'Insolubilia' and Their Solution By Symbolic Logic", Russell solved this paradox by offering a new substitutional theory without the ontology of general propositions. But the resulting substitutional system was too weak to recover arithmetic. Though he entertained the idea of retrofitting the original substitutional theory with orders of general propositions, Russell came to abandon the substitutional theory in favour of *Principia*. I offer new logical axioms to recover arithmetic within substitution in "Logicism's 'Insolubilia' and Their Solution by Russell's Substitutional Theory", in Godehard Link, ed., *One Hundred Years of Russell's Paradox: Papers from the Munich Centenary Conference* (Berlin and New York: de Gruyter, forthcoming).

that build type distinctions into formal grammar. It led Russell to his 1905 theory of denoting,¹⁹ the syntactic approach to the paradoxes of the 1905–08 substitutional theory of propositional structure, the approach to the paradoxes of *Principia*, and as we shall see, to logical atomism itself.

Unfortunately, the doctrine has been widely misunderstood by those influenced by van Heijenoort's characterization of Frege and Russell as holding a thesis that logic is a language which contains its own meta-theory and semantics.²⁰ Van Heijenoort was concerned to distinguish Frege and Russell's conceptions of logic from the Boolean and Tarskian approach which he thought could be defined as construing logic as an "un-interpreted algebra". For instance, the law

$$a + b = b + a$$

can be interpreted to be a law concerning arithmetic addition, with a and b ranging over numbers; or it can be interpreted as a law of union, with a and b ranging over sets; or indeed it can be interpreted as logical disjunction with a and b interpreted as sentences. Similarly, Boolean sums can be interpreted as unions of sets of individuals, or as existential quantification over individuals. Van Heijenoort has quantification theory in mind, and he takes Frege's quantification theory as a genuine language whose variables range over everything. But this entirely misunderstands Frege's reason for thinking his *Begriffsschrift* is informative in a way that an algebra (together with its interpretations) cannot be. The informativity of the *Begriffsschrift* does not lie in its being a genuine language whose variables range over everything. What makes Frege's *Begriffsschrift* informative and distinct from an algebra (together with interpretation) is its adoption of a comprehension principle for functions. Frege's *Begriffsschrift* was not merely a quantification theory, it permits the comprehension of new functions. This is the heart of Frege's logicism. Functions are logical entities, and it is by the comprehension of functions that logical structures are revealed to be contained in what

¹⁹ For a discussion of the origins of the theory of definite descriptions, see Landini, "On Denoting' against Denoting", *Russell*, n.s. 18 (1998): 43–80.

²⁰ Jean Van Heijenoort, "Logic as Calculus and Logic as Language", *Synthese*, 17 (1967): 324–30.

otherwise appear as uniquely arithmetic principles. In this way, Frege's logicism offers a formal uninterpreted calculus which at the same time contains a logical content that transcends what can be accomplished by interpretations of an algebra with fixed operation signs.

The situation with Russell is no different. Indeed, in *Principia*, Whitehead and Russell explicitly observe that they have set out a formal calculus (algebra) together with an intended interpretation.²¹ Russell's doctrine of the unrestricted variable does not hold that logic contains its own semantics. Expressed in modern words, the doctrine is this:

*Any formal calculus for pure logic must adopt only one style of individual or entity variables.*²²

This same doctrine is vehemently advocated by Quine.²³ Only individual variables are part of a proper calculus for logic.

Unfortunately, Quine concludes that a proper calculus for logic must be first-order predicate logic. On Quine's view, predicate variables, class variables, and the like are introduced as part of theories of entities (universals, classes) outside of logic proper. The variables are readily avoided by the adoption of predicates germane to the theory in question—for instance, we have “ Cx ” for being a class, “ $x \in y$ ” for x is a member of y , “ Px ” for x is property or relation, exemplification predicates, and so on. Indeed, Quine maintains that even Russell's type-theory succumbs. Instead of special variables for types, Quine would have “ x is of type (o)” and “ x is of type ((o))” where x is the one and only style of variable. But Quine entirely misses Russell's eliminativism. Russell sought to *solve* the paradoxes plaguing logicism by finding eliminativistic analyses which reveal logical structures. The predicate “ x is of type (o)” is a pseudo-predicate. There is no theory of types of *entities*. The structure of types,

²¹ *PM*, I: 115. Indeed, they explicitly endorse Huntington's independence proofs for the axioms of his algebra of classes. See *PM*, I: 205.

²² In Russell's *Principles* variables of formal implication (quantification theory) are explained by appeal to denoting concepts such as “every term”, where “term” is used synonymously with “logical subject”, “entity”, and “being”. The doctrine of the unrestricted variable is expressed as the doctrine that “whatever is, is one”, and “whatever is, has being”. See *PoM*, pp. 43f., 38, 91, 132.

²³ W. V. O. Quine, “On Carnap's Views On Ontology”, in *The Ways of Paradoxes and Other Essays* (Cambridge, Mass.: Harvard U. P., 1976), pp. 203–11.

not a theory of typed entities, is to be built into an eliminativistic analysis couched in a type-free calculus for logic which adopts only one style of genuine variables. Russell states his method in the following:

... the range of significance must be somehow given with the variable; this can only be done by employing variables having some internal *structure* for such as are to be of some definite logical type other than individuals... But then we have to assume that a single letter, such as x , can only stand for an individual; and that can only be the case if individuals are really all entities, and classes, etc., are merely a *façon de parler*.²⁴

In Russell's view, logicism can be saved from the paradoxes if mathematics is reconstructed within a "no-attributes" (no propositional functions) and "no classes" theory. There are universals (properties and relations in intension). Indeed, in the substitutional theory the horseshoe sign stands for the logical relation of "implication". But logic must proceed without the assumption that every open wff comprehends a universal or a class. Russell achieved this in his substitutional theory by demonstrating how the use of type-indexed predicate variables (and thereby a type-theory of classes and relations-in-extension) can be syntactically recovered in a type-free substitutional theory of propositional structure. Only (type-free) individual variables are allowed in the syntax of the theory. Proxies for predicate variables with type indices are recovered by the structures of the substitutional language. Classes are not identified with (ontologically reduced to) any entities whatsoever of the substitutional ontology.

At times Russell spoke of his treatment of classes as if it offered a meaning analysis of the statements of the naïve theory of classes, showing that class expressions of ordinary language, like definite descriptions, are not referential expressions. But properly speaking Russell was advocating what Kuhn later called a "paradigm shift". The major successes obtained by appeal to the existence of classes, the positive constructions of Cantor, Dedekind, Weierstrass, and Frege, are to be retained within substitution. Russell explained that "... the principles of mathematics

²⁴ "On 'Insolubilia' and Their Solution by Symbolic Logic", in Russell, *Essays in Analysis*, ed. D. Lackey (London: Allen and Unwin, 1973), p. 205. The paper was originally published as "Les paradoxes de la logique", *Revue de Métaphysique et de Morale*, 14 (1906): 627–50.

may be stated in conformity with the theory”, and the theory “... avoids all known contradictions, while at the same time preserves nearly the whole of Cantor’s work on the infinite”.²⁵ The results obtained by appeal to the existence of classes are conceptualized in an entirely new way within substitution. There will be some loss—some flotsam—such as Cantor’s transfinite ordinal number ω_ω , but this loss is to be measured against the successes of the new substitutional constructions. The substitutional theory reveals that Russell’s logicist programme was eliminativistic and not ontologically reductive. It is a programme of structural realism.

This programme extends to *Principia* itself. *Principia* espouses an eliminativistic approach. The logical particles are now statement connectives—there being no ontology of propositions. Classes are not reduced to (identified with) any entities of the ontology of *Principia*, but the structure of a theory of classes is reconstructed nonetheless. Unfortunately, this point is easy to miss. In the substitutional theory matters were clear. There are only individual variables. Type structures are emulated by multiple substitutions of entities. In *Principia* one finds predicate variables adorned with order/type indexes and a discussion of what it calls “propositional functions”. Consider the following passage from “Mathematical Logic as Based on the Theory of Types” (1908):

The difficulty which besets attempts to restrict the variable is, that restrictions naturally express themselves as hypotheses that the variable is of such or such a kind, and that, when so expressed, the resulting hypothetical is free from the intended restriction.... Thus a variable can never be restricted within a certain range if the propositional function in which the variable occurs remains significant when the variable is outside that range. But if the function ceases to be significant when the variable goes outside a certain range, then the variable is *ipso facto* confined to that range, without the need of any explicit statement to that effect. This principle is to be borne in mind in the development of logical types.... [The variable is *internally limited* by its range of significance.]²⁶

Russell hopes to solve the paradoxes plaguing logicism by building structure into variables. To be sure, *Principia*’s account of variables and their

²⁵ “On ‘Insolubilia’ and Their Solution by Symbolic Logic”, p. 213.

²⁶ Russell, “Mathematical Logic as Based on the Theory of Types”, *American Journal of Mathematics* (1908); *LK*, p. 73.

“internal limitations” is perplexing. Following Quine, it is natural to interpret the predicate variables of *Principia* objectually—as ranging over universals (attributes, properties and relations in intension). On this view, *Principia* offers a type- and order-regimented theory of universals.²⁷ Every well-formed formula that is type- and order-stratified comprehends a universal. Bernard Linsky disagrees. Calling attention to certain passages of *Principia*, he maintains that propositional functions are distinct from its universals, and both are genuine entities in spite of Russell’s official position that the former is a “logical fiction”.²⁸ Others follow Church in ignoring *Principia*’s abandonment of propositions, and maintain propositional functions are intentional entities that contain ontological counterparts of quantifiers and logical connectives—they are entities whose values are propositions.²⁹ On my view, *Principia* does not make a distinction between *entities* which are propositional functions and those that are universals. It does not make such a distinction because it does not offer a *theory* of type- and order-stratified intentional *entities* called “propositional functions”. *Principia* was designed to show how logic can proceed to the development of pure mathematics *without* the assumption that every open formula comprehends an intensional entity. Russell had concluded that the paradox (of predication) shows that logic cannot provide any principles to determine what universals there are.

The predicate variables of *Principia* come with order/type indices, but they are given an informal nominalistic semantics. They are not, therefore, “genuine” variables. Only the individual variables of the work are genuine. At the time of *Principia*, Russell held that (as a matter of contingent fact) we are acquainted with universals, and he regarded universals as logical entities which subsist necessarily (if you will). Universals are part of the multiple-relation theory—a theory which forms the base of

²⁷ See Nino Cocchiarella, “The Development of the Theory of Logical Types and the Notion of a Logical Subject in Russell’s Early Philosophy”, *Synthese*, 45 (1980): 71–115.

²⁸ Bernard Linsky, *Russell’s Metaphysical Logic* (Stanford: CSLI Publications, 1999), Chap. 2.

²⁹ Warren Goldfarb, “Russell’s Reasons for Ramification”, in C. Wade Savage and C. Anthony Anderson, eds., *Rereading Russell: Essays in Bertrand Russell’s Metaphysics and Epistemology*, Vol. 12 of *Minnesota Studies in the Philosophy of Science* (Minneapolis: U. of Minnesota P., 1989), pp. 24–40. See also Peter Hylton, *Russell, Idealism and the Emergence of Analytic Philosophy* (Oxford: Clarendon P., 1990).

a nominalistic semantics for *Principia's* predicate variables. But universals are not values of the predicate variables of the system. Indeed, *Principia's* ontology is type-free. *Principia's* individual variables range over all entities—including universals. The work attempts to reconceptualize a ramified type-theory of intensional entities within a type-free theory whose ontology consists of universals, particulars and facts. In short, type and order indices on the predicate variables of the language of *Principia* are explained away as semantic “limitations built into the conditions of significance” of the use of the predicate variables.³⁰ In Russell's view, *Principia* had built types and orders into logical grammar.

Russell's logical atomism was precisely a conception of philosophy as eliminativistic analysis, reconceptualization, and reconstruction. The ontology of an old theory is abandoned (or obviated). Only its structures of the old theory are recovered (when possible). Russell holds that knowledge of physics is confined to structural and mathematical properties, not intrinsic natures of physical substances.³¹ In “Logical Atomism”, Russell is more clear about the nature of the programme. The method advocates a form of structural realism, for it *retains* only the structures given by the laws of the old ontological framework (just as Maxwell's equations for electromagnetic waves in an aether are retained in Einstein's no-aether theory of relativity). The breadth of Russell's eliminativism became wider when he adopted a form of physicalism late in 1918, which he described as “neutral monism”. Indeed, the notion of “acquaintance” as a dyadic relation between a mind and a universal or a sense-datum, so dear to Pears's characterization of Russell's logical atomism, was already abandoned by Russell in 1918. Both the physical theory of matter (i.e., continuants that persist through space-time) and the psychological theory of conscious states (and acquaintance or “noticing” as Russell later called it) are entirely reconceptualized. Only the structures of the old theory, not the ontology, are recovered in the supplanting theory of space-time events and their relationships. Russell's often neglected book *Philosophy* makes his commitment to eliminativism unequivocal.

Once Russell's eliminativistic methods in the substitutional theory

³⁰ See *Russell's Hidden Substitutional Theory*.

³¹ Russell, *Philosophy* (New York: Norton, 1927), p. 157; *AMa*, pp. 264, 287.

and in *Principia* are recognized, a new perspective emerges on Russell's philosophy in general. Russell's method involves an investigation of logical form, but not as an "enumeration" or inventory of logical entities grasped by acquaintance (or logical intuition) coupled with a logical construction. Pears's empiricist characterization of Russell's logical atomism, and its distance from Wittgenstein's Tractarian atomism, ignores Russell's eliminativism. Russell's logical atomism was a programme of eliminativistic analysis and reconstruction—a philosophical method of restructuring that dissolves philosophical conundrums by an ontological elimination which builds logical structure into syntax (and semantics of variables). One must not conflate, as Pears does, Russell's programme of logical atomism with any of the several theoretical constructions he couched within it—be it *Principia's* theory of classes, the multiple-relation theory of judgment, Russell's 1914 construction of physical continuants from physical sense-data, the theory of acquaintance, or neutral monism (which abandons acquaintance as a dyadic relation).³² The theoretical constructions come and go; the programme of atomism remains.

Russell's Apprentice

Russell's philosophical programme of logical atomism was to establish the quest for *logical form* as the centrepiece of a new scientific philosophy. He took his programme to be exemplified by the achievements of mathematicians³³ such as Frege³⁴ on the notion of cardinal number, Cantor on infinity and continuity, Dedekind on the notion of irrationals, and Weierstrass on the notion of the "limit" of a function. Russell writes: "Continuity had been, until he [Cantor] defined it, a vague word, convenient for philosophers like Hegel, who wished to introduce metaphysical muddles into mathematics" (*HWP*, p. 829). The new constructions arithmetizing analysis revealed that it is order, not magnitude,

³² So entrenched is the view that Russell's atomism was committed to an empiricist principle of acquaintance (as a dyadic relation) that some writers even suggest that Russell "converted" to neutral monism and abandoned logical atomism. See Robert Tully, "Three Studies of Russell's Neutral Monism", *Russell*, n.s. 13 (1993): 7.

³³ Russell, "Mathematics and the Metaphysicians" (as "Recent Work in the Philosophy of Mathematics", *The International Monthly*, 1901), in *ML; Papers* 3: 59–74.

³⁴ Russell, *OKEW*₃, p. v.

that is basic to continuity. Similarly, Weierstrass banished the use of infinitesimals in the calculus, showing that the old quantitative notion of a “limit” of a function—i.e., as a number to which other numbers in a series generated by the function approximate as nearly as one pleases—should be replaced by a quite different *ordinal* notion. Russell's examples include metrical geometry's purging the notions of motion and superposition from the properly *metrical* notion of congruence, and even Einstein on space-time. Russell wrote: “Physics, as well as pure mathematics, has supplied material for the philosophy of logical analysis.... What is important to the philosopher in the theory of relativity is the substitution of space-time for space and time” (*ibid.*, p. 833).

The examples that Russell took to be paradigmatic of work in the theory of *logical form*, show that interpretations of Russell mistakenly took the analysis of logical form to provide a *meaning* analysis that reveals the true logical grammar of a *statement* that is hidden by its misleading surface grammar. Quite clearly the work of Weierstrass, Cantor, Frege, and Einstein are not analyses of the ordinary meanings of such notions as limit, continuity, cardinal number, congruence, or space and time. The fundamental idea underlying Russell's science of logical form is not properly characterized as one of a meaning analysis of a *statement*. It is rather one of an eliminativistic analysis, one involving a “paradigm shift”.

For example, eighteenth- and nineteenth-century physics and chemistry offered a number of subtle fluid and aether theories that were highly successful at explaining a wide variety of phenomena. In the process of theory change, the research programmes that gave rise to such theories were supplanted by atomistic physical theories couched within new research programmes. Empirical and conceptual problems pertaining to the aether (such as its elasticity) were dropped, and an entirely new research programme, with a new language and a new set of empirical and conceptual techniques, was inaugurated. Many successes of the earlier aether theories were retained by the theories of the new research programme. Retention, however, may be only partial; the confirmed predictions of an earlier theory in a rival research tradition do not always survive into the supplanting research tradition. Indeed, theoretical processes and mechanisms of earlier theories are at times treated as flot-

sam.³⁵ The supplanting tradition may come to regard the terms of the earlier theories as non-referential, or regard the earlier ontologies as idle wheels that serve no explanatory purpose. This is precisely the sense in which Russell viewed his programme of philosophy as a quest for *logical form*.

Russell advocated a “scientific method in philosophy”, where the “supreme maxim of all scientific philosophizing” is to be this: “Whenever possible, logical constructions are to be substituted for inferred entities.”³⁶ We now see that the supreme maxim is not one of ontological reduction, but one of eliminativistic reconstruction. “This method”, Russell goes on, “so fruitful in the philosophy of mathematics, will be found equally applicable in the philosophy of physics ...” (*ibid.*). Inspired by advances in mathematics, he contended that “every philosophical problem, when it is subjected to the necessary analysis and purification, is found to be not really philosophical at all, or else to be, in the sense in which we are using the word, logical” (*OKEW*₃, p. 35).

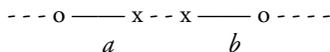
Russell’s eliminativism reveals the unity of Russell’s philosophy of logical atomism and the atomism of Wittgenstein’s *Tractatus*. One must not saddle Russell’s programme of logical atomism with any particular theory he couched within it—be it the 1914 construction of physical objects from sense-data and sensibilia, *Principia*’s theory of classes, or his neutral monism of the 1920s. Russell’s programme of atomism was not committed to empiricism, with sense-data as the fundamental atoms, nor indeed to any other particular ontology. It was committed only to an ideal eliminativistic reconstruction which solves philosophical problems and recovers the successful structures of the earlier abandoned frameworks.

Wittgenstein was Russell’s protégé. He was enthralled by the many successes of Russell’s eliminativist programme, which made logical analysis, followed by logical synthesis (construction), the essential task of philosophy. In reading the *Tractatus*, it is essential to keep in mind that Wittgenstein *accepted* Russell’s eliminativism as part of his own programme. Consider, for instance, his discussion of Kant’s problem of incongruous counterparts:

³⁵ See Larry Laudan, *Progress and Its Problems* (Berkeley: U. of California P., 1977).

³⁶ “On the Relation of Sense-Data to Physics”, *ML*, p. 155; *Papers* 8: 11.

Kant's problem about the right and the left hand, which cannot be made to coincide, exists even in two dimensions. Indeed it exists in one-dimensional space



in which two congruent figures, *a* and *b*, cannot be made to coincide unless they are moved out of this space. The right hand and the left hand are in fact completely congruent. It is quite irrelevant that they cannot be made to coincide.

A right-hand glove could be put on the left hand, if it could be turned round in four-dimensional space. (*TLP* 6.36111)

Concerning this argument Fogelin writes: "This is one of the few arguments in the *Tractatus* that strikes me as just awful. It is surely obvious that Kant's central point is that a right-hand glove and a left-hand glove cannot be made to coincide in a three dimensional space. For this reason he calls them incongruent. Here it will not help to offer—as Wittgenstein does—an alternative definition of congruency."³⁷ Fogelin wholly misses the fact that Wittgenstein's cryptic remark is simply lifted from Russell's discussion of the matter in *The Principles of Mathematics* (pp. 417ff.).

Russell regarded Kant's famous discussion of incongruent counterparts (i.e., that it is a synthetic and yet necessary truth that a right-hand glove cannot be made to coincide spatially with a left-hand glove) as relying upon the erroneous importation of ordinary language notions of superposition and motion into the properly *metrical* notion of congruence—as if congruence requires that there must be a continuous series of equal figures moved through physical space and leading from *A* to *B*. Russell wrote:

No motion will transform *abcd* into a tetrahedron metrically equal in all respects, but with the opposite sense. In this fact, however, there seems, to my mind, to be nothing mysterious, but merely a result of confining ourselves to three dimensions. In one dimension, the same would hold of distances with opposite senses; in two dimensions, of areas. It is only to those who regard

³⁷ Robert Fogelin, *Wittgenstein*, 2nd ed. (London: Routledge & Kegan Paul, 1987), p. 90.

motion as essential to the notion of metrical equality that right and left-handedness form a difficulty.... (PoM, pp. 418ff.)

Fogelin is correct that motion and superposition are certainly part of the ordinary conception of *congruence* used by Euclid, Kant, and others, but this is irrelevant to its reconceptualization (its new “definition”) within the new logicist research programme.

Wittgenstein’s borrowing from Russell on Kant’s incongruous counterparts speaks loudly enough. But there are many more examples. Following von Wright, it is often noted in discussions of the *Tractatus* that Wittgenstein likely came upon the picture theory in his reading of Hertz’s 1894 book, *Principien der Mechanik*.³⁸ In the preface to his book, Hertz suggests that physics constructs mathematical models (*Bilder*) of reality, representing the essential features of the physical world by the relations that hold in the model. Wittgenstein says that a picture must have the same numeric multiplicity as its fact, and Hertz says that a system that is the model of another must satisfy the condition “that the number of co-ordinates of the first system is equal to the number of the second.” The isomorphism between linguistic picture and fact is akin, as Wittgenstein says, to the isomorphic relations between the gramophone record, the musical idea, the score, and the sound-waves (*TLP* 4.014, 4.0141). Similarly, Hertz says that the relation of a dynamical model to the system of which it is regarded as the model is precisely the same as the relation of the images which our mind forms of things to the things themselves. The many similarities are striking.³⁹ Wittgenstein himself invites us to compare his comments with those of Hertz on dynamical models (*TLP* 4.04). But what is often missed is the important connection between Wittgenstein’s philosophy and the purposes that Hertz set himself in his work. Russell discussed Hertz’s work in *The Principles of Mathematics* and took it as an example of conceptual analysis. Working within a kinematic theory that embraces the aether and Maxwell’s equations for electromagnetism, Hertz offers a systematic reconceptualization of Newtonian dynamics that takes “time”, “space”, and “mass” as fundamental and eliminates Newtonian force or energy,

³⁸ Heinrich Hertz, *The Principles of Mechanics* (New York: Dover 1956).

³⁹ See James Griffin, *Wittgenstein’s Logical Atomism* (Seattle: U. of Washington P, 1964), p. 100ff.

thereby avoiding “action at a distance”. The “picture theory” (and the notion of a dynamical model) was employed by Hertz as an aid in achieving this reconceptualization and was not regarded as a *general* account of thought and representation. Hertz’s work offers an example of an eliminativistic approach to conceptual analysis. Indeed, the influence that Hertz’s mechanics supposedly had on Wittgenstein’s Tractarian “picture theory” is often touted, but there is a deeper influence that derives from Russell’s work. Hertz’s reconstruction of mechanics without the concept of “force” is an example of Russellian eliminativism.

Russell’s science of logical form involves the supplanting of one language and ontology by another within which the successes of the former have been retained but completely reconceptualized. Wittgenstein is endorsing Russell’s eliminativistic programme—his quest for logical form. Enthusiastic and brash, Wittgenstein maintained that the perfection of Russell’s programme consists in revealing that one can, in principle, formulate an *ideal* language for science in which *all* logical (and semantic) notions are *shown* by formal grammar. Philosophical problems and conundrums are generated by taking notions such as “identity”, “universal”, “particular”, “complex”, “fact”, “truth”, “falsehood”, “necessity”, “possibility”, “belief”, and non-extensional contexts generally, as if they were primitive. But all logical (and semantic) notions are pseudo-concepts. Russell had made a start, but had not gone far enough.

Our discussion therefore leads to the following thesis:

Wittgenstein’s Tractatus was a handbook of constructive criticisms and preliminary ideas toward the perfection and completion of the Russell eliminativistic programme for a new philosophy of logical form.

This thesis, I contend, is consistent with Wittgenstein’s famous doctrine of showing. It may be recalled that in a letter to Russell of 19 August 1919, Wittgenstein wrote that the doctrine of showing is the “main contention” of the *Tractatus*:

Now I’m afraid you haven’t really got hold of my main contention, to which the whole business of logical propositions is only corollary. The main point is the theory of what can be expressed (*gesagt*) by propositions—i.e., by language—(and, which comes to the same, what can be *thought*) and what can not be expressed by propositions, but only shown (*gezeigt*); which, I believe, is the cardinal problem of philosophy. (*Cambridge Letters*, p. 124)

Of course, the *Tractatus* states its “fundamental idea” as the view that the logical constants are not representatives; that there can be no representatives of the logic of facts (*TLP* 4.0312). Our present thesis is the result of maintaining that these two should be identified. The doctrine of showing is precisely the idea that the logical constants are not representatives.

The possibility of this identification has been largely missed.⁴⁰ It must be understood that Wittgenstein’s notion of the “logical constants” includes much more than the statement connectives, quantifiers, and identity sign of predicate logic. He includes every “formal concept”, every concept that has logico-semantic content. Intensional contexts such as “necessity” and “possibility”, and intentional contexts such as “knows” and “believes” (which are tied up with notions of semantic content and representation), are thereby included. The Tractarian theses of truth-functional compositionality, extensionality, picturing, independence (of atomic statements), and the like, are direct consequences of the doctrine of showing. Wittgenstein’s doctrine of showing demands that all logical and semantic notions (all “logical constants”) are pseudo-predicates that must be shown in the syntax of the ideal theory by its variables.

In fact, the doctrine of showing is but an extension of Russell’s own eliminativistic methods which proclaimed that solutions of the paradoxes facing logicism are to be found by constructing “variables with structure”. Recalling Russell’s passages on “building structure into variables”, it is instructive to note similar remarks in the *Tractatus*. Wittgenstein wrote:

We can now talk about formal concepts, in the same sense that we speak of formal properties.

(I introduce this expression in order to exhibit the source of the confusion between formal concepts and concepts proper, which pervades the whole of traditional logic.)

When something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead it is shown in the very sign for this object. (A name shows that it signifies an object, a sign for a num-

⁴⁰ See Richard McDonough, *The Argument of the Tractatus* (New York: SUNY P, 1986). McDonough makes a start, but he construes the logical constants too narrowly and so attempts to derive the main Tractarian results from truth-functionality alone.

ber that it signifies a number, etc.)

Formal concepts cannot, in fact, be represented by means of a function, as concepts proper can.

For their characteristics, formal properties, are not expressed by means of functions.

The expression for a formal property is a feature of certain symbols.

So the sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept.

So the expression for a formal concept is a propositional variable in which this distinctive feature alone is constant. (*TLP* 4.126)

The notion of a “formal concept” is just the notion of a concept that contains logical or semantic content, and such concepts are pseudo-concepts whose content should be shown in the structure of the variables (of an ideal theory). Wittgenstein continues:

Thus the variable name “ x ” is the proper sign for the pseudo-concept *object*.

Whenever the word “object” (“thing”, etc.) is correctly used, it is expressed in conceptual notation by a variable name.

For example, in the proposition, “There are 2 objects which ...”, it is expressed by “ $(\exists x, y) \dots$ ”.

Wherever it is used in a different way, that is as a proper concept-word, nonsensical pseudo-propositions are the result.

So one cannot say, e.g., “There are objects”, as one might say “There are books”. And it is just as impossible to say, “There are 100 objects”, or, “There are \aleph_0 objects”.

And it is nonsensical to speak of *the total number of objects*.

The same applies to the words “complex”, “fact”, “function”, “number”, etc.

They all signify formal concepts, and are represented in conceptual notation by variables, not by functions or classes (as Frege and Russell believed). (*TLP* 4.1272)

In the *Tractatus* Wittgenstein makes some bold attempts at eliminativistic constructions that would build formal concepts into the variables. Most of the attempts offer little more than the postulation that constructions can be found—they fail to exemplify the “honest toil” that Russell lauded over “theft”. But they are undeniably Russellian. There was only one programme of logical atomism, and it was Russell’s eliminativism.

“Sub Specie Aeternitatis”

Now Pears has maintained that the doctrine of showing is “a baffling doctrine bafflingly presented”.⁴¹ “If we try to deduce the meaning of the doctrine and its justification from the things that he puts on the list of what can be shown and not said”, says Pears, “we do not get much help. For the list includes such disparate items as the existence of a named object (*TLP* 4.1211, 5.535), the identity of the world with my world (*TLP* 5.62), and the value of anything that has value (*TLP* 6.4–6.421)”.⁴² Objectors to our identification of showing with the Tractarian “fundamental idea” and with Russellian eliminativism will immediately seize upon Wittgenstein’s discussion of solipsism, God, aesthetics and value in the *Tractatus*. Indeed, in his early attempts to publish the *Tractatus*, Wittgenstein wrote to von Ficker proclaiming that “the book’s point is an ethical one”.⁴³ This may seem to present an insuperable challenge to the present interpretation. Though ethical truths would appear to be necessary, ethical concepts certainly do not seem to contain logico-semantic content. But on Wittgenstein’s view, all necessity is logical necessity; and there is ample evidence that Wittgenstein was entertaining the thought that ethics and logic are kindred spirits. In his *Notebooks 1914–1916* we find the comment: “Ethics does not treat of the world; ethics must be a condition of the world, like logic” (p. 83).

It is worth observing, in this regard, that Russell was very interested in Spinoza’s *Ethics*. In 1912 Russell published a paper entitled “The Essence of Religion”—a piece excerpted from a failed book project called *Prisons* which Russell wrote in 1911 while his affair with Ottoline Morrell was new. Let me quote from Russell’s outline of the first chapter:

Religion consists in union with the universe. Formerly, union was achieved by assimilating the universe to our own conception of the Good . . . we must find a mode of union which asks nothing of the world, and depends solely upon ourselves. . . .

The moralist divides the world into good and bad. . . . But besides this dualistic attitude, there is another, wholly compatible with it, but monistic: an attitude which ignores the differences between the good and the bad, and loves all alike. This is the essence of religion; but because it has not been clearly distinguished

⁴¹ *The False Prison*, I: 143.

⁴² *Ibid.*

⁴³ G. H. von Wright, *Wittgenstein* (Oxford: Basil Blackwell, 1981), p. 83.

from the moralist's attitude, it has been supposed, wrongly, to require the belief that the world is good.... Every such demand [that the world shall conform to our standards] is an endeavour to impose Self upon the world.... The essence of religion is union with the universe achieved by subordination of the demands of Self. This subordination is not complete if it depends upon a belief that the universe satisfies some at least of the demands of Self.⁴⁴

Self-interest and subjectivity are a "prison" in Russell's view, because they shut out the possibility of impersonal contemplation of the world—a Spinozistic "intellectual love of God" interpreted as an attitude produced by viewing the world "*sub specie aeternitatis*".⁴⁵ According to "The Essence of Religion", the "infinite self" is universal and impartial, and peace comes to this self through *harmony* or *agreement* with the whole, by means of its experience of "wisdom". Russell approvingly calls this "mysticism", though he warns that it is misguided to interpret it as a "perception of new objects". It is, instead, "a different way of regarding the same objects, a contemplation more impersonal, more vast, more filled with love, than the fragmentary, disquiet consideration we give to things when we view them as a means to help or hinder our own purposes."⁴⁶

Of course, Russell reported that Wittgenstein "detested" "The Essence of Religion". Russell wrote to Ottoline that in Wittgenstein's view he "had been a traitor to the gospel of exactness and wantonly used words vaguely; also that such things are too intimate to print." But contrary to Monk's biographical account (1: 280), this does not show that he disagreed with Russell's Spinozism. Quite the contrary, in his *Notebooks 1914–1916*, Wittgenstein wrote that "the good life is the world seen *sub specie aeternitatis*.... The thing seen *sub specie aeternitatis* is the thing seen together with the whole of logical space" (p. 75; see also *TLP* 6.45). Wittgenstein goes on to say that: "In order to live happily I must be in agreement with the world. And that is what 'being happy' means" (p. 77). It is significant, indeed, that Wittgenstein's comments on ethics and God in the *Notebooks* and the *Tractatus* reflect Russell's Spinozistic

⁴⁴ "Prisons 1" (Morrell Papers, U. of Texas at Austin); *Papers* 12: 105; Kenneth Blackwell, *The Spinozistic Ethics of Bertrand Russell* (London: Allen and Unwin, 1985), pp. 140–1.

⁴⁵ Blackwell, p. 160f.

⁴⁶ Russell, "The Essence of Religion", *Hibbert Journal*, Oct. 1912; *Papers* 12: 114.

theme of a contemplation of the facts of the world *sub specie aeternitatis*. The proper ethical attitude is harmony or peace of mind with the world no matter what are its facts, and this is shown by the conspicuous absence of any statements concerning what is good or evil, or obligatory. Such statements would reflect a self in disharmony with the world's facts. On this view, a logically ideal language would exclude the formation of ethical statements—their inability to be formulated itself shows that the proper ethical perspective is that of harmony. Wittgenstein's comment to von Ficker that the *Tractatus's* point is ethical and his adoption⁴⁷ of the Spinozistic title *Tractatus Logico-Philosophicus*, reflect his agreement with (and “perfection” of) Russell's Spinozistic ideas on ethics.

The Dark Side of Radical Eliminativism

Russell's eliminativism became, in the hands of the youthful Wittgenstein, the doctrine of showing. But Russell's eliminativism was originally *moderate*; it exempted logical notions and knowledge of logic from undergoing eliminativistic analyses. Wittgenstein hoped to extend Russell's eliminativism radically. In the language of the logically ideal scientific theory, a language in which the analysis of logical form is complete, all (and only) logico-semantic notions would be built into logical syntax. Russell's science of logical form involves the supplanting of one language and ontology by another within which the successes of the former have been retained but completely reconceptualized. The supplanting framework, of course, does not exist at present. It is an ideal, a limit of scientific and conceptual analysis, obtainable in principle. The idea of picturing, borrowed from Hertz's work, is enlisted to point the direction that a completed eliminativistic analysis would take. For Wittgenstein, the completed framework does not itself contain any “theory” of conceptual analysis or conceptual change. The toil of eliminativistic analysis and its justification are thrown away once the supplanting scientific framework is adopted. This is important to Wittgenstein's vision, for according to his radical eliminativism, all philosophical problems of ontology are to vanish at the limit of scientific investigation.

⁴⁷ Correspondence shows that this title was suggested by Moore (von Wright, *Wittgenstein*, p. 97). The original title was *Logische-Philosophische Abhandlung*.

In light of these important connections, we can see that it is very misleading to characterize Wittgenstein's Tractarian philosophy as being antithetical to Russell's conception of philosophy as the quest for logical form. Wittgenstein's conception of logical form was of a piece with Russell's notion of an eliminativistic analysis, and is not properly characterized as an account of representation in thought and language. Hacker writes:

One deep misapprehension runs through Russell's introduction [to the *Tractatus*]: namely, that Wittgenstein was concerned with elaborating conditions for a logically perfect language. This was no trivial misunderstanding, since the point of the book was to elaborate the logico-metaphysical conditions for any possible language. It was a treatise on the essential nature of any form of representation whatever—a metaphysics of symbolism.⁴⁸

Russell's interpretation is the correct one. The *Tractatus* aims to make steps toward a Russellian eliminativist analysis and reconstruction of the logico-semantic notions employed in ordinary (and technical) languages.

To be sure, Wittgenstein calls attention to ordinary language when he writes that “we need to understand the logic of our language”, and that “all the propositions of our everyday language, just as they stand, are in perfect logical order ...” (*TLP* 5.5563). These passages have been used to support the thesis that Wittgenstein was concerned with the formal conditions for representation in thought in *any* possible language. But passages such as these are properly understood in the context of the retentive nature of the eliminativistic programme that Wittgenstein shared with Russell. The retentiveness explains why Wittgenstein says that one must understand the logic of our language. For the successes of the old paradigm must be understood, reconceptualized, and recovered, in the new paradigm. The new analysis explains what the old use of the expressions was trying to get at, and reveals the source of its successes and failures. Properly put, when Russell speaks of “the logical form of a statement” he is envisioning the supplanting of one linguistic framework (and theory) by another in such a way as to retain (wherever possible) the successes of the old framework; and through a reconceptualization, to reveal the sources and solutions of the philosophical conundrums the

⁴⁸ Wittgenstein's Place in Twentieth-Century Analytic Philosophy, p. 69.

former theory generated. So it was with Wittgenstein's *Tractatus*. The intent is to reconstruct all logico-semantic notions. The explanatory successes of the use of such notions in ordinary language (and some technical languages) is to be retained and explained in the new research programme. It is precisely in this retentive form of eliminativistic reconstruction that Wittgenstein meant that "ordinary language is in perfect logical order", that "all philosophy is a critique of language", and that we need to "understand the logic of our language".

It is all too easy to seize upon passages of the *Tractatus* which appear to be antithetical to Russell's programme, exploiting them as if they undermine any argument that Wittgenstein shared Russell's programme of logical form. Tractarian doctrines do destroy some of Russell's famous constructions. For example, with Wittgenstein's demand of extensionality, there is no longer any reason for Russell's contextual definition of class symbols. Moreover, with Wittgenstein's demand that the predictable nature of a universal can only be shown by keeping predicate expressions in predicate positions, Russell's multiple-relation theory (the foundation of the recursive definition of "truth" and "falsehood" used to philosophically explain *Principia's* hierarchy of orders) is undermined.⁴⁹ But it must be understood that it was because Wittgenstein was working toward the *perfection* of Russell's programme⁵⁰ that Russell was enticed by Wittgenstein's Tractarian musings—undeveloped though they were.⁵¹

⁴⁹ See Landini, "A New Interpretation of Russell's Multiple-Relations Theory of Judgment", *History and Philosophy of Logic*, 12 (1991): 37–69.

⁵⁰ Wittgenstein's letter to Russell of March 1919 corroborates my claim that the *Tractatus* was a work in alliance with Russell. Wittgenstein wrote: "I believe I've solved our problems finally. But it upsets all our theory of truth, of classes, numbers and all the rest." See *Auto.*, 2: 116.

⁵¹ It was with just this attitude that Russell attempted to investigate some of Wittgenstein's ideas in his new introduction for the second edition of *Principia* (1925). In the striking fashion of eliminativism, he sketched a reconstruction showing, as he put it, "that the principle of extensionality is not shown to be false, when strictly interpreted, by the analysis of such sentences as 'A believes *p*.'" Moreover, he showed that mathematical induction could be proved without *Principia's* reducibility principle by modifying its grammar and adopting Wittgenstein's extensionalist position that a "function can only occur in a proposition through its values". See Landini, "The Definability of the Set of Natural Numbers in the 1925 *Principia Mathematica*", *Journal of Philosophical Logic*, 25 (1996): 597–615.

Wittgenstein's Tractarian ideas of truth-functionality and extensionality, his construal of logical necessities as tautologies, his attempt to obviate logical deduction in favour of decidability, his suggestion that identity is part of quantification, his picture theory, and even his solipsism,⁵² are thoughts about what the supplanting ideal eliminativist language for empirical science would look like.

Russell's eliminativism was originally *moderate*. It exempted logical notions and knowledge of logic from undergoing eliminativistic analyses. Wittgenstein hoped to extend Russell's eliminativism to logic itself. Indeed, the most important part of Wittgenstein's "perfection" of Russell's eliminativistic programme, though it is one which Russell hardly could believe viable, was that knowledge of logic (and semantics) must itself undergo eliminativistic reconstruction. Knowledge of logic is not properly *knowledge* at all. With *all* logico-semantic notions construed as pseudo-concepts, Wittgenstein hoped to avoid Russell's reliance on a faculty of logical intuition which acquaints us with logical forms such as "universal", "particular", "fact", "complex", "negation", "disjunction", and "generality". On Wittgenstein's view, there are no such primitive notions and Russell's 1913 suggestion that there is a relation of logical intuition (logical acquaintance) with logical objects is misguided.⁵³ Philosophy, as Russell held, is properly the eliminativistic quest for logical form. But in Wittgenstein's perfection of Russell's eliminativistic programme, there is no "science" of logical form.

In his introduction to the *Tractatus*, Russell observed that Wittgenstein's radical elimination of *all* logical and semantic notions is self-undermining. If *all* logical and semantic notions are pseudo-concepts, then the philosopher will be at a loss to offer any argument on behalf of the eliminativistic constructions. The strictly correct method would be to speak in the ideal eliminativistic language of the new science, and observe a silence which shows one's refusal to use the pseudo-predicates that perpetuate philosophical conundrums. This threatens to collapse

⁵² His "solipsism" suggests that the ideal scientific language would embody a form of neutral monism—a thesis he and Russell discussed, and that Russell came to embrace in 1918. See John W. Cook, *Wittgenstein's Metaphysics* (Cambridge: Cambridge U. P., 1994).

⁵³ In his abandoned 1913 book, *Theory of Knowledge*, Russell held that we have acquaintance with the logical forms "negation" and "disjunction". But this is *not* to say that the logical particles stand for relations.

Russell's eliminativistic programme of logical form into a mysticism whose "truths" can only be rendered by oracular pronouncement. Russell couldn't accept this, and in his introduction to the *Tractatus* he voiced his opposition to the mysticism into which showing collapses.⁵⁴ Wittgenstein's now famous response was to offer the simile of throwing away a ladder once climbed. Russell was not satisfied, and the idea has won few adherents. We may recall Ramsey's quip: "But what we can't say we can't say, and we can't whistle it either."⁵⁵

Wittgenstein scholars will concede that the Tractarian theses of picturing, extensionality, and decidability are false. But many maintain, nonetheless, that Tractarian criticisms of Russell's conception of logic and philosophy are telling. Hacker heralds the *Tractatus* as "a colossal advance over ... nineteenth-century thought and over the philosophical ideas of Frege and Russell".⁵⁶ To be sure, the modern conception of logic rejects Russell's conception and makes logic a branch of the mathematical study of formal systems. But Wittgenstein's criticisms of Russell had no hand in this, and his own conception of logic as an ineffable ontological "scaffolding" of the world was far more out of step with the modern mathematical conception of logic and mathematics than was Russell's. The story that Wittgenstein's *Tractatus* "revolutionized" the philosophy of logic and shattered Russell's programme in philosophy is simply a myth.⁵⁷

⁵⁴ The situation is not unlike the dilemma that eliminativistic materialism faces. The very data of mental states, beliefs, sensations, etc., which is to be explained, is undermined by the eliminativism.

⁵⁵ Frank Ramsey, "General Propositions and Causality", in *The Foundations of Mathematics*, ed. R. B. Braithwaite (New York: Harcourt, Brace, 1931), p. 238.

⁵⁶ *Wittgenstein's Place in Twentieth-Century Philosophy*, p. 34.

⁵⁷ *Ibid.*, pp. 37, 81, 278.
