COMPLETING RUSSELL'S LOGIC

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The epsilon calculus improves upon the predicate calculus by systematically providing complete individual terms. Recent research has shown that epsilon terms are therefore the "logically proper names" Russell was not able to formalize, but their use improves upon Russell's theory of descriptions not just in that way. This paper details relevant formal aspects of the epsilon calculus before tracing its extensive application not just to the theory of descriptions, but also to more general problems with anaphoric reference. It ends by contrasting a Meinongian account of cross-reference in intensional constructions with the epsilon account.

INTRODUCTION

In Russell's theory of definite descriptions there are, it will be remembered, three clauses: with "the King of France is bald" these are "there is a king of France", "there is only one king of France" and "he is bald". Russell used an iota term to symbolize the definite description, but it is not an individual symbol: it is an "incomplete" term, as he explained it, since "the King of France is bald" is taken to have the complex analysis,

$$(\exists x)(Kx \,.\, (y)(Ky \supset y = x) \,.\, Bx)$$

and so it does not have the elementary form "Bx". Russell hypothesized that, in addition to the linguistic expressions gaining formalizations by means of his iota terms, there was another, quite distinct class of expressions, which he called "logically proper names". Logically proper names would, amongst other things, take the place of the variable in such forms as "Bx". Russell suggested that demonstratives might be in this class, but he could give no further formal expression to them.

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Hilbert and Bernays, in their *Grundlagen der Mathematik*, introduce a kind of complete symbol, by contrast with Russell, defending what would later be called "presuppositional theory" of definite descriptions. The first two clauses of Russell's definition,

$$(\exists x)(Kx \,.\, (y)(Ky \supset y = x))$$

are not taken, by presuppositionalists, to be part of what is asserted by "the King of France is bald"; they are, instead, the conditions under which one is allowed to introduce into the language an individual term for "the King of France", which then satisfies the matrix of the quantificational expression above, and becomes a proper symbol to replace the variable in such expressions as "Bx". Hilbert and Bernays still used an iota term for this purpose, although it is quite different from Russell's iota term, since, when it is part of the language, it is equivalent to the related epsilon term. It has been realized, more recently, that epsilon terms, being complete symbols, are the "logically proper names" Russell was looking for, and that their natural reading is indeed as forms of demonstratives.

It is at the start of Book 2 of the *Grundlagen* that Hilbert and Bernays introduce epsilon terms. They first go on to produce a theory of non-definite descriptions of the same presuppositional sort to their theory of definite descriptions. Thus they permit an eta term to be introduced into the language if the first of Russell's conditions is met, "$(\exists x)Kx$"; this term then satisfies the associated matrix, but it is, in general, an individual, presuppositional term of the same kind as their iota one. There is a singular difference in certain cases, however, since the presupposition of the eta term can be proved conclusively, for certain matrices. Thus we know, for any predicate "F", that

$$(\exists x)((\exists y)Fy \supset Fx)$$

since this is a theorem of the predicate calculus. The eta term this theorem permits us to introduce is what Hilbert and Bernays call an epsilon term. Thus we get the epsilon axiom

$$(\exists y)Fy \supset F\varepsilon xFx$$

which therefore implies

$$(\exists y)Fy \equiv F\varepsilon xFx.$$

So an epsilon term is very unlike the generality of eta terms, since its introduction is clearly not dependent on any contingent facts about *F*. It is this which permits completely formal theories using epsilon terms to be developed, because such epsilon terms, unlike Hilbert and Bernays' iota terms, are always defined, and, as the equivalence indicates, they refer to exemplars of the property in question. The above predicate calculus theorem, in other words, provides the existence condition for certain objects, which the various epsilon calculi then go on to symbolize reference to, using epsilon terms. Copi has explained the theorem's relation with exemplars very fully (1973, p. 110).

Kneebone read epsilon terms as formalizing indefinite descriptions (1963, p. 101), and this idea is commonly also found in the work of his pupil, Priest, although strangely Priest himself has pointed out that reading "$(\exists x)(Gx \,.\, Fx)$" as "$G\varepsilon xFx$" will not do.[1] Hilbert read the epsilon term in the above case "the first *F*", which indicates its place in some, otherwise unspecified, well-ordering of the *F*'s—for instance, in connection with arithmetical predicates, that generated by the least number operator. So "εxFx" is not "an *F*". Moreover, as Copi's discussion makes very clear, it is possible that an epsilon term refers to something which is in fact not *F*—it does this, of course, if there are no *F*'s at all—and that will lead us to theories of reference which materialized only in the 1960s and later, when reference came to be properly distinguished from attribution. If there are *F*'s then the first *F* is a chosen one of them; but if there are no *F*'s then "the first *F*" must be non-attributive, and so denotes something it cannot connote. It functions like a Millian name, in other words, with no applicable sense. With denotation in this way clearly distinguished from description we can then start to formalize the cross-reference which even Russell needed to link his first two conditions "there is one and only one king of France" with his further condition "he is bald". For, by an extension of the epsilon equivalent of the existential condition, the "he" in the latter comes to be a pronoun for the same epsilon term as arises in the former—whether or not the former is true. And such anaphoric cross-reference in fact may stretch into and across intentional contexts of the kind Russell was also concerned with, such as "George IV wondered whether the author of

[1] Priest 1979, p. 6; see also Slater 1988, p. 285.

Waverley was Scott". For, of course, he was indeed Scott, and we may all now know very well that he was Scott. So we obtain a formalization for transparency in such locutions.

That puts developed epsilon calculi at variance with Fregean views of intensional contexts—and also the Kripkean semantics which has continued to support Frege in this area. But Fregean intensional logic did not incorporate Millian symbols for individuals, and in particular, as we shall see in detail later, that meant it could not clearly distinguish individuals from their identifying properties. The addition of epsilon terms provides the facility for separating, for instance,

$$s = \varepsilon x(y)(Ay \equiv y = x)$$

and

$$(y)(Ay \equiv y = s)$$

and so for isolating the proper object of George IV's thought.

DESCRIPTIONS AND IDENTITY

When one begins to investigate the natural-language meaning of epsilon terms, it is significant that Leisenring, writing in 1969, merely notes the "formal superiority" of the epsilon calculus, comparing some of its pedagogic features with the comparable ones in the predicate calculus (1969, p. 63). Apparently its main value, in Leisenring's day, was that it could prove all that was provable in the predicate calculus but in a smarter, and less tedious, way. Epsilon terms, for Leisenring, were just clever calculating instruments.

Evidently there is more to the epsilon calculus than this, but until more recent times only the natural-language meaning of the above epsilon axiom has been dwelt upon. There are a couple of further theorems within the epsilon calculus, however, which will show its extended range of application: they are about the nature and identity of individuals, as befits a calculus which systematically provides a means of reference to them.

The need to provide logically proper names for individuals only became generally evident some while after Russell's work on the theory of descriptions. The major difficulty with providing properly referential terms for individuals, in classical predicate logic, is what to do with "non-denoting" terms, and Quine, following Frege, simply gave them an ar-

bitrary, though specific, referent. The approach was formalized perhaps most fully by Kalish and Montague, who gave the two rules:

$$(\exists x)(y)(Fy \equiv y = x) \;/\; F\iota xFx$$
$$\neg\,(\exists x)(y)(Fy \equiv y = x) \;/\; \iota xFx = \iota x \neg\,(x = x)^{2}$$

where, in explicitly epsilon terms, we would have

$$\iota xFx = \varepsilon x\,(y)(Fy \equiv y = x).$$

Kalish and Montague were of the opinion, however, that their second rule "has no intuitive counterpart, simply because ordinary language shuns improper definite descriptions" (*ibid.*, p. 244). And certainly, in that period, the revelations which Donnellan (1966) was to publish about non-attributive definite descriptions were not well known. But ordinary language does not, we now know, avoid non-attributive definite descriptions, although their referents are not as constant as Kalish and Montague's second rule requires. In fact, by being improper their referents are not fixed by semantics at all: like demonstratives, the referents of logically proper names are found only in their pragmatic use. Stalnaker and Thomason were more appropriately liberal with their complete individual terms. And these referential terms also had to apply, they knew, in every possible world.[3] But a fuller coverage of identity and descriptions, in modal and general intensional contexts, is to be found in Routley, Meyer and Goddard (1974), and also Hughes and Cresswell (1968). With these Australasian thinkers we find the explicit identification of definite descriptions with epsilon terms.[4]

Which further theorems in the epsilon calculus are behind these kinds of identification? There is one theorem in particular which demonstrates strikingly the relation between Russell's attributive, and some of Donnellan's non-attributive, ideas (see Slater 1988). For

$$(\exists x)(Fx \,.\, (y)(Fy \supset y = x) \,.\, Gx)$$

is logically equivalent to

[2] Kalish and Montague 1964, pp. 242–3.
[3] Thomason and Stalnaker 1968, p. 363.
[4] E.g., Hughes and Cresswell 1968, p. 203.

$$(\exists x)(Fx \,.\, (y)(Fy \supset y = x)) \,.\, Ga$$

where $a = \varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x))$. For the latter is equivalent to

$$Fa \,.\, (y)(Fy \supset y = a) \,.\, Ga$$

which entails the former. But the former is

$$Fb \,.\, (y)(Fy \supset y = b) \,.\, Gb$$

with $b = \varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x) \,.\, Gx)$, and so entails

$$(\exists x)(Fx \,.\, (y)(Fy \supset y = x))$$

and

$$Fa \,.\, (y)(Fy \supset y = a).$$

But then, from the uniqueness clause,

$$a = b$$

and so

$$Ga$$

making the former entail the latter.

The former expression, as we have seen, encapsulates Russell's theory of descriptions, in connection with "the *F* is *G*"; it involves the explicit assertion of the first two clauses, to do with the existence and uniqueness of an *F*. A presuppositional account like that in Hilbert and Bernays, which was later popularized by Strawson, would not involve the direct assertion of these two clauses: on a presuppositional account they form the precondition without which "the *F*" cannot be introduced into the language. But both of these accounts forget the use we have for non-attributive definite descriptions. Since Donnellan (and see Slater 1963), we now know that there are no preconditions on the introduction of "the *F*"; and "the *F* is *G*", as a result, may always be given a truth value. Hence "*Ga*" properly formalizes it. If the description is non-attributive, i.e. if the first two clauses of Russell's account are not both true, then the referent of "the *F*" is simply up to the speaker to nominate.

But one detail about Donnellan's actual account must be noted at this point. He was originally concerned with definite descriptions which were

improper in the sense that they did not uniquely describe what the speaker took to be their referent. And on that understanding the description might still be "proper" in the above sense—if there still was something to which it uniquely applied. Specifically, Donnellan would originally allow "the man with martini in his glass" to refer to someone without martini in his glass whether or not there was some unique man with martini in his glass. But someone talking about "the man with martini in his glass" can be rightly taken to be talking about who this phrase describes, if it does in fact describe someone—Devitt and Bertolet pointed this out in criticism of Donnellan.[5] It is this latter part of our linguistic behaviour which the epsilon account of definite descriptions respects, for it permits definite descriptions to be referring terms without being attributive, but only so long as nothing has the description in question. Hence it is not the first quantified statement above, but only, so to speak, the third part of it extracted which makes the remark "the *F* is *G*".

This becomes plain when we translate the two statements using relative and personal pronouns:

There is one and only one *F*, which is *G*.
There is one and only one *F*; it is *G*.

For "it" here is an anaphoric pronoun for "the (one and only) *F*", and it still has this reference even if there is no such thing, because that is just a matter of the grammar of the language. Now the uniqueness clause is required for two such statements to be equivalent—without it there would be no equivalence, as we shall see—and that means that the relative pronoun "which" is not itself equivalent to the personal pronoun "it". So it was because Russell's logic could not separate the (bound) relative pronoun from the (unbound) personal pronoun that it could not formulate the logically proper name for "it", and instead had to take the whole of the first expression as the meaning of "the *F* is *G*". Using just the logic derived from Frege, it could not separate out the cross-referential last clause.

But how can something be the one and only *F* "if there is no such thing"? This is where a second theorem in the epsilon calculus is relevant:

[5] Devitt 1974, Bertolet 1980.

$$[Fa \,.\, (y)(Fy \supset y = a)] \supset [a = \varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x))].$$

For the singular thing is that this entailment cannot be reversed, so there is a difference between the left-hand side and the right-hand side, i.e. between something being alone *F*, and that thing being the one and only *F*. We get from the left-hand side to the right-hand side once we see that the left-hand side entails

$$(\exists x)(Fx \,.\, (y)(Fy \supset y = x))$$

and so

$$F\varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x)) \,.\, (z)(Fz \supset z = \varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x))).$$

By the uniqueness clause we get the right-hand side. But if we substitute "$\varepsilon x\,(Fx \,.\, (y)(Fy \supset y = x))$" for "$a$" in the whole implication, then the right-hand side is necessarily true. But the left-hand side is then equivalent to

$$(\exists x)(Fx \,.\, (y)(Fy \supset y = x))$$

which is, in general, contingent; hence the implication cannot be logically reversed.

The difference is not available in Russell's logic. In fact Russell confused the two forms, since he formalized possession of an identifying property by using the identity sign

$$a = \imath x Fx$$

making it appear that some, maybe even all, identities are contingent. But all proper identities are necessary, and it is merely associated identifying properties which are contingent. Ironically, Frege used a complete term for definite descriptions in his extensional logic, as was mentioned before. But Russell explicitly argued against the arbitrariness of Frege's definition, in the case where there isn't just one *F*, when setting up his alternative, attributive theory of descriptions in "On Denoting". Had Frege's complete term been more widely used, and, for a start, been used in his intensional logic, results like those above might have been better known earlier.

Hughes and Cresswell, at least, appreciated that in addition to "contingent identities" there were also necessary identities, and differentiated

between them as follows:

> Now it is contingent that the man who is in fact the man who lives next door is the man who lives next door, for he might have lived somewhere else; that is, *living next door* is a property which belongs contingently, not necessarily, to the man to whom it does belong. And similarly, it is contingent that the man who is in fact the mayor is the mayor; for someone else might have been elected instead. But if we understand ["the man who lives next door is the mayor"] to mean that the object which (as a matter of contingent fact) possesses the property of being the man who lives next door is identical with the object which (as a matter of contingent fact) possesses the property of being the mayor, then we are understanding it to assert that a certain object (variously described) is identical with itself, and this we need have no qualms about regarding as a necessary truth. This would give us a way of construing identity statements which makes $[(x = y) \supset L(x = y)]$ perfectly acceptable: for whenever $x = y$ is true we can take it as expressing the necessary truth that a certain object is identical with itself.
> (Hughes and Cresswell 1968, p. 191)

There is more hanging on this matter, however, than Hughes and Cresswell appreciated. For now that we have the logically proper names, i.e. complete symbols, to take the place of the variables in such expressions as "$x = y$", not only do we see better where the contingency of the properties of such individuals comes from—just the linguistic possibility of improper definite descriptions—we also see, contrariwise, why constant epsilon terms must be rigid. It is because identities involving such terms are necessary.

Frege, for instance, thought that we could not derive "*a* believes the Morning Star is illuminated by the sun" from "*a* believes the Evening Star is illuminated by the sun", even though the Morning Star is the Evening Star. But,[6] from "$BaI\varepsilon xEx$" we can derive "$BaI\varepsilon xMx$", if $\varepsilon xEx = \varepsilon xMx$; what we cannot derive is "$Ba(\exists x)[(y)(My \equiv y = x) \,.\, Ix]$" from "$Ba(\exists x)[(y)(Ey \equiv y = x) \,.\, Ix]$", even if $(\exists x)(\exists y)(Mx \,.\, Ey) \,.\, (x)(y)[(Mx \,.\, Ey) \supset x = y]$. Russell improved matters somewhat by distinguishing a primary, transparent sense "$(\exists x)[(y)(Ey \equiv y = x) \,.\, BaIx]$" from the secondary, opaque sense "$Ba(\exists x)[(y)(Ey \equiv y = x) \,.\, Ix]$", since the former, with $(\exists x)(\exists y)(Mx \,.\, Ey) \,.\, (x)(y)[(Mx \,.\, Ey) \supset x = y]$, does entail "$(\exists x)[(y)(My \equiv y = x) \,.\, BaIx]$". But without epsilon terms to provide

[6] See, for instance, Slater 1992.

explicit instantiations of the primary-sense forms, Russell was in no position to detach their second conjuncts.

THE EPSILON CALCULUS' PROBLEMATIC

It follows that there is no essential grammatical difference between such an intensional anaphoric remark about someone's mind, as "The ancients believed there was a star in the morning which was illuminated by the sun. But it was a planet", i.e.

$$Ba\,(\exists x)(Mx \,.\, Ix) \,.\, P\varepsilon xBa\,(Mx \,.\, Ix)$$

and the extensional cross-reference, for instance, in "There was a man in the room. He was hungry", i.e.

$$(\exists x)Mx \,.\, H\varepsilon xMx.$$

What has been the problem, fundamentally, has been getting the cross-reference formalized first of all in the purely extensional kind of case. Yet this just requires extending the epsilon replacement for an existential statement, by means of a repetition of the associated epsilon term, as was mentioned before with respect to "he" in Russell's case. The only difference in the intensional case is that, to obtain the required cross-referencing one must move from "$Ba\,(\exists x)(Mx \,.\, Ix)$" to "$(\exists x)Ba\,(Mx \,.\, Ix)$" via "$Ba\,(Mb \,.\, Ib)$" with $b = \varepsilon x\,(Mx \,.\, Ix)$ to get a public referential phrase for the object. And note that, while the required epsilon term "$\varepsilon xBa\,(Mx \,.\, Ix)$" is then defined intensionally, it still refers to a straightforward extensional object—the planet Venus, of course.

It is now better understood how the epsilon calculus allows us to do this.[7] The starting point is the possibility illustrated in the theorem about Russellian definite descriptions above, of separating out what otherwise, in the predicate calculus, would be a single sentence into a two-sentence piece of discourse, leaving the existence and uniqueness clauses in one place, and putting the characterizing remark in another. The point really starts to matter when there is no way to symbolize in the predicate calculus some anaphorically linked remarks where there is no uniqueness

[7] See Slater 1986, Purdy 1994, Egli and von Heusinger 1995, and Meyer Viol 1995, Chap. 6, for instance.

clause, as in the above extensional case. This is what became a problem for the Fregean and Russellian logicians who woke up to the need to formalize anaphoric reference in the 1960s.

It can be seen, as before, how it was lack of the epsilon calculus which was the major cause of the difficulty. Thus Geach (1962, p. 126), in an early discussion of the issue, went to the extremity of insisting that there could be no syllogism of the following form:

A man has just drunk a pint of sulphuric acid.
Nobody who drinks a pint of sulphuric acid lives through the day.
So, he won't live through the day.

Instead, Geach said, there was only the existential conclusion:

Some man who has just drunk a pint of sulphuric acid won't live through the day.

Certainly one can only conclude

$$(\exists x)(Mx \,.\, Dx \,.\, \neg\, Lx)$$

from

$$(\exists x)(Mx \,.\, Dx)$$

and

$$(x)(Dx \supset \neg\, Lx)$$

within Fregean predicate logic. But one can still conclude

$$\neg\, L\varepsilon x\,(Mx \,.\, Dx)$$

within its conservative extension: Hilbert's epsilon calculus.

And through inattention to that extension, Geach was entirely stumped later, when he discussed his famous intensional example (3):

Hob thinks a witch has blighted Bob's mare, and Nob wonders whether she (the same witch) killed Cob's sow, (Geach 1967, p. 628)

i.e.

$$Th\,(\exists x)(Wx \,.\, Bxb) \,.\, OnK\varepsilon x Th\,(Wx \,.\, Bxb)\,c.$$

For he saw this could not be his (4):

$$(\exists x)(Wx \,.\, ThBxb \,.\, OnKxc)$$

or his (5):

$$(\exists x)(Th(Wx \,.\, Bxb) \,.\, OnKxc).$$

But a reading of the second clause as

Hob wonders whether the witch who blighted Bob's mare killed Cob's sow
[*cf.* Geach's (18)]

in which "the witch who blighted Bob's mare killed Cob's sow" is analyzed in the Russellian manner, as Geach's (20):

Just one witch blighted Bob's mare and she killed Cob's sow

does not provide the required cross-reference—for one thing because of the uniqueness clause then involved. Of course the descriptive replacement for the personal pronoun "she" in the Hilbertian expression, namely "what Hob thinks is a witch that blighted Bob's mare", does not have any implication of uniqueness.

The inappropriateness of the uniqueness clause in Russellian analyses has been widely discussed. However it did not deter Neale, who much later wrote a whole book defending a largely Russellian account of definite descriptions, and cross-sentential anaphora. But he got no further with Geach's case above than proposing that "she" might be localized to "the witch we have been hearing about" (1990, p. 221), thinking in general that definite descriptions merely should be relativized to the context. But a greater change is needed than that. For it is not that, in addition to witches in the actual world, there are also witches in people's minds, but merely that, in addition to witches in the actual world, there are things in the actual world which are thought, or believed to be, witches. Geach's "the same witch" is also inappropriate, on the same grounds.

A large amount of the important, initial work in this area was done by another person very influenced by the Russellian tradition: Evans. But Evans also explicitly separated from Russell over the matter of uniqueness, for instance in connection with back-reference to a story about a man and a boy walking along a road one day:

One does not want to be committed, by this way of telling the story, to the existence of a day on which just one man and boy walked along a road. It was

with this possibility in mind that I stated the requirement for the appropriate use of an E-type pronoun in terms of having answered, or *being prepared to answer upon demand*, the question "He? Who?" or "It? Which?"

In order to effect this liberalization we should allow the reference of the E-type pronoun to be fixed not only by predicative material explicitly in the antecedent clause, but also by material which the speaker supplies upon demand. This ruling has the effect of making the truth conditions of such remarks somewhat indeterminate; a determinate proposition will have been put forward only when the demand has been made and the material supplied.

(Evans 1977, pp. 516–17)

It was Evans who popularized the name "E-type pronoun" for the pronoun in such cases as "A Cambridge philosopher smoked a pipe and he drank a lot of whisky", i.e.

$$(\exists x)(Cx \,.\, Px) \,.\, D\varepsilon x(Cx \,.\, Px).$$

He also argued at length, in line with the above (*ibid.*, p. 516), that what was distinctive about E-type pronouns was that such a conjunction of statements as this was not equivalent to "A Cambridge philosopher, who smoked a pipe, drank a lot of whisky", i.e.

$$(\exists x)\, Cx \,.\, Px \,.\, Dx).$$

Obviously the epsilon account supports this, since the contrast illustrates the point remarked before: only the expression which contains the relative pronoun can be symbolized in the predicate calculus, since to symbolize the personal pronoun its epsilon extension is needed.

Some grammarians have tried to handle this sort of issue in intensional contexts by returning to Meinongian "intensional objects", or the "counterparts" of actual individuals in alternative worlds. For example, Saarinen considers the following case:

Bill believes that the lady on the stairs [is acquainted with] him, but John knows she is only a wax figure. (Saarinen 1978, p. 277)

About this Saarinen says, "Both of the attitudes are of the wax lady, and yet all the relevant individuals in the doxastic worlds are not wax ladies but human beings' (*ibid.*, p. 282). However, as before, in addition to human beings in this world it is not that there are also human beings in

people's minds—merely, also, that there are things in this world which are taken to be human beings, by people. Saarinen supports his judgment with a Russellian, i.e. attributive, reading of "the lady on the stairs", but more important is his retention of the Meinongian idea that such an intensional object as the golden mountain has to be made of gold, and even that an impossible intensional object like the round square has to still be both round and square. Thereby, of course, Saarinen misses the possibility that what Bill believes is the one and only lady on the stairs is not really a lady on the stairs. And it is this possibility, exactly, which allows it to be just the plain, and everyday, physical wax figure which is the object of both Bill's and John's attitudes. The form of Saarinen's case, if "the lady on the stairs" is attributive, is

$$Bb\,(\exists x)((y)(Ly \equiv y = x)\,.\,Abx)\,.\,KjW\varepsilon xBb\,((y)(Ly \equiv y = x)\,.\,Abx)$$

but the second conjunct of this entails

$$\neg\, L\varepsilon xBb\,((y)(Ly \equiv y = x)\,.\,Abx)$$

since knowledge entails truth, and being a wax figure entails not being a lady.

As was mentioned before, it is even contingent that *the lady on the stairs* is a lady on the stairs, but the source of this non-doxastic, and simply modal, contingency, which allows the same object to appear in other possible worlds, cannot be properly seen until we link it with the linguistic possibility of improper, i.e. non-attributive, definite descriptions. Seeing the source of this even more radical contingency is thus essentially linked to seeing how there can be *de re* attitudes. But it is also directly linked with the much more substantial programme of replacing such a metaphysical view as Meinong's simply with accurate linguistic analysis: "philosophical problems arise through misconceptions of grammar", to paraphrase Wittgenstein.

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