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Optimal Strategy Hand-rank Table for Jacks or Better, Double Bonus, and Joker Wild

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Optimal Strategy Hand-rank Table for Jacks or Better, Double Bonus, and Joker Wild Video Poker

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Abstract

Video poker is a casino game based on five-card draw poker played on a computerized console. Video poker allows players an opportunity for some control of the random events that determine whether they win or lose. This means that making the right play can increase a player's return in the long run. For that reason, optimal strategy hand-rank tables for various types of video poker games have been recently published and established to help players improve their return (Ethier, 2010).

Ethier posed a number of open problems in his recent book, *The Doctrine of Chances: Probabilitistic Aspects of Gambling* among which were some in video poker. In this thesis we consider the most popular video poker games: Jacks or Better, Double Bonus, and Joker Wild. Ethier produced an optimal strategy hand-rank table for Jacks or Better. We expand on his method to produce optimal hand-rank tables for Double Bonus,and Joker Wild. The method involves enumerating all possible discards, computing the expected returns, and then finding a way to rank them according to optimal discard based on the payoffs. The methodology combines combinatorics with probability and C++ programming. For Double Bonus and Joker Wild new terminologies are introduced in order to illustrate how diffferent cards can affect the magnitude of expected return of a particular hand. Furthermore, ranks whose organization and specification are noteworthy in the hand-rank tables are examined and provided with in-depth explanations and supporting examples. The final result is a hand-rank table for use by the player. The player chooses that discard which ranks highest in the hand-rank table corresponding to potential final hands.

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Chapter 1

Introduction

1.1 History of Video Poker

The term gambling is used to describe games of chance that satisfy an individual's desire and fantasy to wager a fixed amount of money in the expectation of a large return. But gambling is not merely a game; it is a pervasive form of sport, entertainment, and even addiction. Furthermore, gambling is currently a major international commercial activity whose market value was estimated to be approximately $335 billons in 2009, and nearly two-thirds of this figure comes from lotteries and casinos (*The Economist*, 2010). Among many gambling games, video poker games are a favourite among players, combining two games in one: poker and slot machines. This marriage gave birth to one of the most popular online and offline games (Tamburin, 2010). In order to understand the development of video poker, one must first understand its history.

The exact origin and the year of the predecessor of video poker are arguable due to different aspects that consider video poker machines adequate such as video monitor, prizes, and payoff schedules (Fey, 2002). In the mid 1970s, as the reality of inexpensive personal computers came to fruition, the gambling industry expanded its territory by embracing this new technology. Si Redd, who was influential in many slot machine innovations, formed a company known as Sircoma (Si Redd's Coin Machines) by whom the electronic version of poker was firmly established in 1979. As mentioned by Paymar (2004), since the manufacturer had not yet fully evaluated the mean expected payout of the game, Redd used payoff schedules, complained to be unfair by public in order to play safe. But, the later addition of a return on a pair of jacks or better enormously increased the popularity of the game. Then, it was Charles August Fey (2002) who added the feature of draw to his popular video poker machine, "Poker Matric." Allowing some of the reels to be held when taking a second spin for the unwanted cards increased the popularity of slot machines to a greater extent.

1.2 How to Play Video Poker

Video poker, like all other slot machines, is easy to play and involves no interaction with a dealer or other players. Typically, a player inserts into the machine one to five units (coins) to place a bet. The highest payoff table is reserved for the maximum bet (five units). Then, cards are electronically dealt from a randomly shuffled deck of standard 52 cards (53 in Joker Wild) and these are displayed on a monitor screen. For each of the five cards, a player then decides which of the five cards to hold or discard. Each of the cards dealt is equally likely to occur. Following a discard, of k cards, k additional cards are then drawn, again with equal probability, from the remaining cards. The player's hand is then evaluated and a payoff returned based on how high the hand ranks. If the hand does not match the minimum rank needed for payoff, then there is no return.

1.3 Misconceptions about Video Poker

Players often tend to overlook the use of optimal strategies. Henry Tamburin (2010), a professional gambler, explained why. The reason is threefold. They are either not aware that such strategy exists, they think that having strategy cards in casino is illegal, or they do not know how to use strategy cards. Tamburin affirmed that pocketsize strategy cards for various types of video poker are now commercially available for different levels of players. Secondly, the use of strategy cards is legal in all gambling jurisdictions. Jurisdictions have not yet addressed the legality of the use of software for gambling, but some jurisdictions have specifically made internet gambling through the application of software illegal. Hence, application of software, not like strategy cards, should be carefully used according to the determined laws of jurisdiction in different states or countries. Thirdly, not knowing or refusing to learn how to use strategy cards is merely an excuse for lack of effort (Tamburin, 2010).

1.4 Reasons for Playing Video Poker

We all might have heard the adage that the house always has the advantage. However, video poker is an exception to that rule as video poker allows something that other slot machines lack – an element of skill. Therefore, players who want to increase their winnings should make an effort to understand the mathematics behind the game. If players have not fully grasped the mathematical concepts, they might fall into the casino's trap thinking that they have a good chance of winning, just by looking at the payoff schedules. This is where players make their first mistake. Having the basic knowledge of probability will allow players to discern games that have positive expectation and consequently increase their chances of winning. If the knowledge of probability is applied, then video poker becomes a rewarding choice for casino players. Games such as roulette, keno, and craps are merely based on luck. However, video poker is one of the games where players are given an opportunity not only to control, but also to change the turn of event based on their discard decision. This means that making the right plays for any given hand can maximize player's return in the long run. Video poker has one of the highest, if not the highest expected returns under optimal play. As a matter of fact, among different types of video poker, more lucrative games like Double Bonus and Joker Wild offer more than 100 percent expected return under optimal play.

1.5 Non-mathematical Factors that Help Players

There are also non-mathematical factors that may help players gain an advantage even though certain games have calculated returns of less than 100 percent, such as Jacks or Better. If players can afford to bet larger amounts on each play, use of slot club can be a crucial catalyst. Slot clubs were invented by casinos to build loyalty among their customers as they offer rooms, meals, travelling expense, and coupons. According to Bob Dancer and Liam Daily (2004), large casinos around the globe offer 0.25 to 0.33 percent cash back to players if they deposit a certain amount of money into their chip card. Besides cash back, the values of complimentaries and promotions provide additional returns. On the other hand, players should avoid any circumstances that have adverse influence on the play of the game. To name a few, noisy environment, consumption of alcohol, medication and psychological factors along with having to pay a tip and tax for a bigger payout can all affect the net gain.

Table 1.1: Complete List of Equivalence Classes at Jacks or Better

five distinct denominations $(a,\ b,\ c,\ d,\ e)$: $\binom{13}{5} = 1{,}287$ ways (include hands ranked no pair, straight, flush, straight flush, royal flush)							
(1, 1, 1, 1, 1)	4	(1, 1, 2, 3, 3)	24	(1, 2, 2, 1, 2)	12	(1, 2, 3, 2, 1)	24
(1, 1, 1, 1, 2)	12	(1, 1, 2, 3, 4)	24	(1, 2, 2, 1, 3)	24	(1, 2, 3, 2, 2)	24
(1, 1, 1, 2, 1)	12	(1, 2, 1, 1, 1)	12	(1, 2, 2, 2, 1)	12	(1, 2, 3, 2, 3)	24
(1, 1, 1, 2, 2)	12	(1, 2, 1, 1, 2)	12	(1, 2, 2, 2, 2)	12	(1, 2, 3, 2, 4)	24
(1, 1, 1, 2, 3)	24	(1, 2, 1, 1, 3)	24	(1, 2, 2, 2, 3)	24	(1, 2, 3, 3, 1)	24
(1, 1, 2, 1, 1)	12	(1, 2, 1, 2, 1)	12	(1, 2, 2, 3, 1)	24	(1, 2, 3, 3, 2)	24
(1, 1, 2, 1, 2)	12	(1, 2, 1, 2, 2)	12	(1, 2, 2, 3, 2)	24	(1, 2, 3, 3, 3)	24
(1, 1, 2, 1, 3)	24	(1, 2, 1, 2, 3)	24	(1, 2, 2, 3, 3)	24	(1, 2, 3, 3, 4)	24
(1, 1, 2, 2, 1)	12	(1, 2, 1, 3, 1)	24	(1, 2, 2, 3, 4)	24	(1, 2, 3, 4, 1)	24
(1, 1, 2, 2, 2)	12	(1, 2, 1, 3, 2)	24	(1, 2, 3, 1, 1)	24	(1, 2, 3, 4, 2)	24
(1, 1, 2, 2, 3)	24	(1, 2, 1, 3, 3)	24	(1, 2, 3, 1, 2)	24	(1, 2, 3, 4, 3)	24
(1, 1, 2, 3, 1)	24	(1, 2, 1, 3, 4)	24	(1, 2, 3, 1, 3)	24	(1, 2, 3, 4, 4)	24
(1, 1, 2, 3, 2)	24	(1, 2, 2, 1, 1)	12	(1, 2, 3, 1, 4)	24		
one pair $(a,\ a,\ b,\ c,\ d)$: $\binom{13}{9,3,1} = 2{,}860$ ways							
(1, 2, 1, 1, 1)	12	(1, 2, 1, 2, 3)	24	(1, 2, 3, 1, 1)	24	(1, 2, 3, 3, 3)	12
(1, 2, 1, 1, 2)	12	(1, 2, 1, 3, 1)	24	(1, 2, 3, 1, 2)	24	(1, 2, 3, 3, 4)	12
(1, 2, 1, 1, 3)	24	(1, 2, 1, 3, 2)	24	(1, 2, 3, 1, 3)	24	(1, 2, 3, 4, 1)	24
(1, 2, 1, 2, 1)	12	(1, 2, 1, 3, 3)	24	(1, 2, 3, 1, 4)	24	(1, 2, 3, 4, 3)	12
(1, 2, 1, 2, 2)	12	(1, 2, 1, 3, 4)	24	(1, 2, 3, 3, 1)	24	(1, 2, 3, 4, 4)	12
two pair $(a,\ a,\ b,\ b,\ c)$: $\binom{13}{10,2,1} = 858$ ways							
(1, 2, 1, 2, 1)	12	(1, 2, 1, 3, 1)	24	(1, 2, 1, 3, 3)	24	(1, 2, 3, 4, 1)	12
(1, 2, 1, 2, 3)	12	(1, 2, 1, 3, 2)	24	(1, 2, 1, 3, 4)	24	(1, 2, 3, 4, 3)	12
three of a kind $(a,\ a,\ a,\ b,\ c)$: $\binom{13}{10,2,1} = 858$ ways							
(1, 2, 3, 1, 1)	12	(1, 2, 3, 1, 4)	12	(1, 2, 3, 4, 4)	4		
(1, 2, 3, 1, 2)	24	(1, 2, 3, 4, 1)	12				
full house $(a,\ a,\ a,\ b,\ b)$: $\binom{13}{1,1,11} = 156$ ways							
(1, 2, 3, 1, 2)	12	(1, 2, 3, 1, 4)	12				
four of a kind $(a,\ a,\ a,\ a,\ b)$: $\binom{13}{1,1,11} = 156$ ways							
(1, 2, 3, 4, 1)	4						

1.6 Equivalence Classes

Equivalence classes are a means to facilitate the computation of expected returns and it is necessary to understand them. We consider each of the player's $\binom{52}{5} = 2{,}598{,}960$ possible initial hands ($\binom{53}{5}$ for Joker Wild), and then determine for each such hand which of $2^5 = 32$ ways to play maximizes the expected return. However, having to consider all the possible ways of playing 2,598,960 hands is both inefficient and even computer intensive. Equivalence classes simplify this effort. An *Equivalence relation* is a relation that partitions a set into multiple disjoint subsets in such a way that every element in a set is a component of one and only one subset of the partition. This means that two elements of the set are "related" by an equivalence relation or *equivalent* if and only if these two elements are in the same subset. These disjoint subsets defined by an equivalence relation are called *equivalence classes.* The same idea is applied in video poker. Two hands are called *equivalent* if they have the same denominations and if they have the same corresponding suits after a permutation of $(\diamondsuit, \clubsuit, \heartsuit, \spadesuit)$ (Ethier, 2010). Within the same subset, where the denominations of two initial hands are the same and each corresponding suit is also the same after a permutation, two equivalent hands have the same optimal strategy providing the same expected return because they are essentially the *same* hands. For instance, A♡-A♠-A♣-2♡-3♠ is equivalent to a hand with the same denominations of different suit A♠-A♣-A♡-2♠-3♣, A♢-A♠-A♣-2♢-3♠. In fact, there are $\binom{4}{3}\binom{3}{1}\binom{2}{1} = 24$ equivalent hands of A♡-A♠-A♣-2♡-3♠. These 24 hands have the same denominations and same corresponding permutation of suits. However, not every hand with a particular set of denominations has 24 equivalent hands. The number of equivalence classes for a particular set of denominations depends on both permutation of suits and types of poker hands. An equivalence class containing A♡-A♠-A♣-2♡-3♢ has $\binom{4}{3}\binom{3}{1}\binom{4-3}{1} = 12$ hands, one containing A♡-A♠-A♣-2♢-3♢ has $\binom{4}{3}\binom{4-3}{1} = 4$ hands and so forth. The bottom line here is that any two hands that belong to the same equivalence class with the same denominations produce the same expected return and so it suffices to consider only one of them. By only taking one representative hand for each set of denominations and permutation of suits, we can reduce the amount of calculation by approximately a factor of 20 ($\simeq 19.32901 = \frac{2{,}598{,}960}{134{,}459}$) because 134,459 different equivalence classes are sufficient enough to represent every combination of hands.

$$\binom{13}{5}51 + \binom{13}{1,3,9}20 + \binom{13}{2,1,10}8 + \binom{13}{1,2,10}5 + \binom{13}{1,1,11}3 = 134,459 \tag{1.1}$$

As a check, we compute the total number of hands by summing the sizes of the equivalence classes using numbers in Table 1.1:

$$1,287(1\cdot4+15\cdot24)+2,860(8\cdot12+12\cdot24)+858(1\cdot4+7\cdot12+5\cdot24)+156(1\cdot4+2\cdot12) = 2,598,960 \tag{1.2}$$

We next introduce the notation in Ethier (2010). If a player is given a particular set of five distinct denominations, we assign each card m_1, m_2, m_3, m_4, m_5 in an increasing order. This implies that the range of these cards is $2 \leq m_1 \leq m_2 \leq m_3 \leq m_4 \leq m_5 \leq 14$, where 14 represents an ace. Of course, by the nature of a deck of cards, m_1 cannot be the same as m_5 because this implies that all five cards have the same denomination. Only the maximum of four cards can be the same. Then, we can correspond each denomination with the suit n_1, n_2, n_3, n_4, n_5 by numbering $n_i \in 1, 2, 3, 4, i = 1, 2, 3, 4, 5$. We assign the smallest available integer for each suit that does not appear in a higher denomination. Thus,

$n_1 = 1, n_2 \leq 2, n_3 \leq \max(1, n_2) + 1, n_4 \leq \max(1, n_2, n_3) + 1, n_5 \leq \max(1, n_2, n_3, n_4) + 1.$ Hence, there is one-to-one correspondence between equivalence classes of a particular hand with denominations $\{m_1, m_2, m_3, m_4, m_5\}$ and corresponding suits $\{n_1, n_2, n_3, n_4, n_5\}$. The complete list of equivalence classes is displayed in Table 1.1 with the size of each equivalence class.

1.7 Objectives and Organization of Thesis

I was exposed to the field of probability in gambling during my undergraduate studies, and I gained a great interest in determining optimal probability bounds of an event in gambling whose calculation and knowledge in applied statistics can be monetarily applicable in real life. Dr. Hoppe suggested a set of open problems listed in *The Doctrine of Chances: Probabilistic Aspects of Gambling*, which his colleague and co-author Dr. Stewart N. Ethier published in 2010. The ones on video poker stood out among many other topics for the reasons that demand and popularity of the game are immense, and also that a hand-rank table under optimal play for Jacks or Better had recently been produced. However, at the best of my knowledge, hand-rank table for Double Bonus and Joker Wild have not been published. Therefore, first understanding and verifying the optimal strategy at Jacks or Better and then generalizing the methodology for Double Bonus and Joker Wild seemed to be a reasonable goal for my research. For Double Bonus and Joker Wild, all the ideas and conventions covered in the paper is the contribution of my work.

This thesis intends to illustrate how applied probability is incorporated behind video poker (especially in Jacks or Better, Double Bonus, and Joker Wild) and to raise an awareness of the importance of fully comprehending the hand-rank table for each game and other miscellaneous elements that are involved if one wants to maximize the expected return in the long run. The study is organized in three main chapters to discuss the hand-rank table for different types of video poker games and to highlight the importance of certain holdings in the hand-rank tables. First, I present the steps to validate each hand-rank table. Then, all possible ranges of each rank in the hand-rank table that affect the expected return are displayed in order to comfirm the order of each rank. Finally, another chapter is included to examine the effect that each rank has on the overall mean expected payout under optimal strategy.

Chapter 2

Methodology

2.1 Overview

Marshall (2006) once published the complete list of all 134,459 equivalence classes of hands in Deuces Wild along with the optimal strategy for each equivalence class. This required 357 pages, seven columns per page, and 55 hands per columns. Not only human errors are often bound to happen for this kind of problem, but it is also needless to say time-consuming. The total number of distinct equivalence classes alone hints that an use of computer is mandatory in this problem. The C++ program Ethier wrote determines for each equivalence class the strategy that maximizes the expected return by computing the conditional expected returns for each of the 32 possible choices of discards, storing this information. Then, it tabulates the overall payout distribution under playing the optimal strategy for each hand. Probabilities are expressed as ratios of integers to make them more readable. Although the computer can print out exactly one specific instruction as to which cards to hold or discard for each of the 134,459 equivalence classes, it is essential to construct a strategy that works for all equivalence classes so that no software is required. This is the strategy or hand-rank table and it needs to be easily read and understood by players without any mathematical background (Tamburin 2010). Players need to be able to interpret the hand-rank table quickly. Dancer and Daily (2004) pointed out how the loss of playing time, not to mention playing wrong hands, can significantly reduce the overall payout based on their personal experiences. Therefore, a hand-rank table should be constructed in a way that players can execute the strategy in short period of time without strong mathemetical backgrounds, yet it has to be precise enough to work for all possible hands.

Dr. Ethier has generously provided me with his C++ computer program code at Jacks or Better that cycles through all the equivalence classes and outputs the mean expected return under optimal play. He also laid out the types of holdings or descriptions of each rank for Jacks or Better. So, with an assumption that these are the complete set of all the ranks, the first objective was to confirm the correctness of the table by checking whether the hand-rank table works for any representative hands that match the description of each rank, whether each rank is in the right order with the respect to expected return, and lastly to determine whether there exists any new ranks that need to be expressed in the hand-rank table. To do so, I modified some of the printing options and added few more lines of code so that if a particular hand is entered, the program would output all 32 conditional expected returns and provide the strategy that has the highest expected return. Then, I compared each hand and ranked them case by case. I needed to manually consider combinations with different expected return within each rank in the hand-rank table and make sure that they exhausted the range of all possible hands (shown in Appendices). For Double Bonus and Joker Wild, I had to write out all the possible holdings case by case. For Joker Wild, another set of equivalence classes due to the addition of a joker was needed, and a way of expressing a joker needed to be defined. After many trials, I learned that some hands within the same rank had the same expected returns, and some had slightly different expected returns due to the different combination of discards. Moreover, I have come to realize that not every discard plays a role in affecting the expected return, but it is different combinations of *penalty cards* that change the expected return. Greater in-depth characteristics about penalty cards will be discussed in Section 3.3.

2.2 Requirement to Validate Hand-rank Tables

Here, I present the steps in the proof required to validate hand-rank table. The exhaustive range of possible hands within each rank is listed and ordered accordingly in the Appendices.

Step 1: Find generic representations of each hand for all ranks and order them from highest to lowest expected return. Check if there are any missing ranks in the hand-rank table. For Double Bonus and Joker Wild where ranks are not yet defined, try to work using the same ranks used in Jacks or Better. Then, considering different combinations of discards and enhancing restrictions on different cards, generate new ranks and rearrange them if necessary (Some exceptions may apply and these cases will be explained later).

Step 2: Find generic combinations of discards (penalty cards) for each hand that produce different expected return in Step 1 (As mentioned earlier, discards of penalty cards are sensitive to expected return, so combinations of different penalty cards should be considered and recorded).

Step 3: If a hand in Step 2 no longer has the same optimal strategy as Step 1, make a new rank.

Step 4: Compute expectations for each value of Step 2 $\times$ Step 3 combinations.

Step 5: Tabulate maximum and minimum from Step 4.

Step 6: Check for overlap between two ranks. If there are any, the strategy that works when both ranks occur in a hand needs to be ranked first. If both ranks cannot appear concurrently, then place the rank with the higher expectation in the absence of penalty cards.

Chapter 3

Jacks or Better

3.1 General Attributes about Jacks or Better

Jacks or Better, often called draw poker, is one of the classic games of video poker. According to Dancer and Daily (2004), it is the easiest of the standard video games to play at a high strategy level, and it is ideal to learn because general principles learned in Jacks or Better will apply to many types of other video poker. As briefly mentioned earlier, for each of the five cards, a player must decide whether to hold or discard each card. If a player discards k cards, he is dealt k new cards from the remaining deck, called the *packing deck*, with each of the $\binom{47}{k}$ combinations of cards equally likely. As the name of the game suggests, any poker hands that have greater values than a pair of jacks or better guarantee a return. However, it is important for players to note that depending on the casino and the amount bet, payoff schedules may vary. The player may see Jacks of Better games described as 9/6, 8/6, 9/5, 8/5, 8/5/35, and 8/5 Bonus, for instance, where first number represents the payoff for for a full house, second for flush, and third for four of a kind, if needed. For instance, 8/5/35 means that a full house pays 8 for 1 (8 units returned for each unit bet), a flush pays 5 for 1, and four of a kind pays 35 for 1. Obviously, the theoretical returns and optimal strategies vary. In this work, only 9/6 Jacks or Better is discussed because it is the most popular type of game.

Table 3.1: Full-pay Jacks or Better payoff odds and pre-draw frequencies – maximum bet

rank	payoff odds	number of ways
royal flush	800 for 1	4
straight flush	50 for 1	36
four of a kind	25 for 1	624
full house	9 for 1	3,744
flush	6 for 1	5,108
straight	4 for 1	10,200
three of a kind	3 for 1	54,912
two pair	2 for 1	123,552
pair of jacks or better	1 for 1	337,920
others	0 for 1	2,062,860
total		2,598,960

3.2 Expected Values

Denoting the return on a unit bet by R, we use expectation $E[R]$ to compare hands. Players should keep in mind that the strategies described below assume that the maximum bet is made, otherwise unfavorable payoff schedules apply (Dancer & Daily). There are many synonyms used in the paper such as "preferable," "better," "more valuable," "superior" to avoid repetition of saying one hand has "higher expected return" than others.

Suppose a player is dealt A♣-Q♢-J♣-T♣-9♣ in any order where holding A♣-J♣-T♣-9♣ and A♣-J♣-T♣ seem be both plausible strategies.

If a player holds the four card flush A♣-J♣-T♣-9♣,

$$E[R_{A,J,T,9}] = 6\left(\frac{9}{47}\right) + 1\left(\frac{6}{47}\right) + 0\left(\frac{32}{47}\right) = \frac{60}{47} \approx 1.27660 \tag{3.1}$$

If a player decides to hold the three-card two-gap royal flush A♣-J♣-T♣ instead,

$$\begin{aligned} E[R_{A,J,T}] &= 800\frac{\binom{2}{2}\binom{45}{0}}{\binom{47}{2}} + 6\frac{[\binom{9}{2} - \binom{2}{2}\binom{7}{0}]\binom{38}{0}}{\binom{47}{2}} + 4\frac{\binom{4}{1}\binom{3}{1}\binom{40}{0} - \binom{2}{2}\binom{45}{0}}{\binom{47}{2}} + 3\frac{3\binom{3}{2}\binom{44}{0}}{\binom{47}{2}} \\ &\quad + \frac{3\binom{3}{1}^2\binom{41}{0}}{\binom{47}{2}} + 1\frac{2\binom{3}{1}\binom{6}{0}\binom{38}{1} + \binom{4}{2}\binom{43}{0} + \binom{3}{2}\binom{44}{0}}{\binom{47}{2}} \\ &= \frac{1,372}{1,081} \approx 1.26920 \end{aligned} \tag{3.2}$$

Since $E[R]$ is used to rank hands, then for this particular example, drawing to a three-card is marginally better or preferable than drawing to a three-card royal flush since $E[R_{A,J,T,9}] > E[R_{A,J,T}]$.

3.3 Penalty Cards

There is an important matter that needs to be addressed. *Penalty cards* refer to discards that reduce the chance of qualifying for certain paying hands in the payoff table. A discard that decreases the chance of attaining a straight is called a *straight penalty card* denoted as *sp*, and likewise a discard that decreases the chance of attaining a flush is called a *flush penalty card* denoted as fp. 9*sp* is a special case of a straight penalty card in which the discard has to be 9 and straight penalty card. Ethier's hand-rank (Table 3.2) for Jacks or Better is delicate enough to be sensitive to *sp*, fp, and 9*sp*. However, he neither shows nor discusses possible combinations of more than one penalty cards in a single hand. Therefore, we felt the need to include these combinations to show the degree at which they diminish the value of expected return.

Consequently, additional terminologies are introduced to enable us to identify different combinations of penalty cards and measure the value of their returns. Discarding a high penalty card (any of J, Q, K, A) is denoted as *hp*. A card that can be both straight penalty card (sp) and high penalty card (hp) is denoted as *shp*. n that comes in front of sp, hp, shp represents the number of sp, hp, shp present in a hand. For organization purpose, I named and categorized sp by the number of possible straights (s) that it can make with the optimal holding. If holding 3-RF, 3-SF, or 3-S is the optimal strategy, sp is named either outside ($s = 2$) or inside ($s = 1$). For example, with 8♣-T♢-J♢-Q♡-K♢, the optimal holding is the three-card royal flush, where there exists a Q♡ straight penalty card. Since T♢-J♢-K♢ along with Q♡ can form two possible straights ($s = 2$) with either 9 or A, Q♡ is defined as outside sp. If Q♡ were replaced with A♡ in this particular hand, A♡ would have been identified as an inside sp because it can form only one possible straight. If holding 2-RF or 2-S is the optimal strategy, sp is also present and named either an outside ($s = 3$), inside ($s = 2$), or double-inside ($s = 1$). In the similar manner, when 1-RF or holding a single card happens to be the optimal strategy, a hand can have outside1 ($s = 4$), outside ($s = 3$), inside ($s = 2$), double-inside ($s = 1$) straight penalty card. As explicitly shown in Appendices, the more number of straights that a penalty card can make, the more costly and detrimentally it can affect the expected return. Thus, the outside sp ($s = 3$) diminishes the value of a hand more severely than inside sp ($s = 2$), and in return more than double inside ($s = 3$). Furthermore, combinations of multiple straight penalty cards can degrade the value of a return even more severely.

Likewise, fp (flush penalty) card reduces the probability that the player will make a flush (including a royal flush which is also a type of flush). A hand such as 8♢-9♣-Q♢-K♢-A♢ whose optimal strategy is to hold Q♢-K♢-A♢ has 8♢ as fp because discarding 8♢ decreases the probability of flush. Even when the optimal strategy is to hold fewer than three cards such as holding J♢-Q♢ in 2♢-5♣-7♠-J♢-Q♢ (optimal strategy is to hold J♢-Q♢), there can exist a fp such as 2♢ in this example. Also, hands like 2♢-5♣-7♡-8♠-K♢ and 2♢-5♢-7♡-8♠-K♢ show that there exists a flush penalty card when the optimal holding is only one card.

3.4 Approach to Validating Jacks or Better Strategy

There may exist varying combinations of discards in the same rank that produce different expected return. So, each rank has to be supported with a proof that any hand that matches the description of particular rank infers the same strategy despite the difference in level of expected return. However, considering all the possible combinations of discards is in fact time-consuming. As briefly mentioned earlier, we learned that discards that affect expected return are different types of straight penalty cards, flush penalty cards, high cards or combinations of them. This means it is sufficient enough to only consider combinations of different types of penalty cards as discards, then I compute the range of expected return within the same rank and check whether they all produce the same strategy. After that, it needs to be made sure that each rank is correctly ranked in a descending order with the respect to their expected returns. By following each step of validation procedures listed in Methodology along with this idea in mind, such a hand-rank table was produced.

3.5 Ordering System and Explaining Exceptions

It was initial, intuitive assumption that each rank is ordered from the highest to lowest and ideally there is no overlap of expected return between two different ranks in Dr. Ethier's hand-rank table (Table 3.2). But, it is not the case. It may seem odd at first to believe the premise that not all the hands need to be ranked according to $E[R]$, yet the table gives the optimal strategy. If a rank A is a subset of rank B (A $\subseteq$ B), in other words, A is wholly contained inside B, then rank A needs to be placed first despite the scale of expected return. Otherwise the rank B (superset) becomes the optimal strategy for all cases that match the description of B. Having to rank 4-F: $AHTx$ + K, Q, J, or T before 3-RF despite the fact that the latter rank has higher expected return is one of the examples in the table. This particular form of 4-F (which is also a form of 3-RF) requires different optimal strategy than other forms of 4-F. Nonetheless, it is intuitive to think that 3-RF always needs to come first before any form of 4-F because the lowest return of 3-RF (9♡-9♠-T♠-Q♠-A♠, E[R] = 1.28677) is strictly higher than the highest return of 4-F: $AHTx$ + K, Q, J, or T (5♠-T♠-Q♡-K♠-A♠) which gives 1.27659 as its expected return. But, if 3-RF is placed first, 4-F: $AHTx$ + K, Q, J, or T will never be given a chance to be played because 4-F: $AHTx$ + K, Q, J, or T is a form of 3-RF (4-F: $AHTx$ + K, Q, J, or T $\subseteq$ 3-RF). So, if 3-RF were to be placed first, the optimal strategy for 4-F: $AHTx$ + K, Q, J, or T will be no longer holding four cards of the same suit, but instead it will be holding three potential royal flush cards, inferring incorrect optimal strategy. It would suggest players to hold T♠-K♠-A♠ instead of 5♠-T♠-K♠-A♠ in the above example.

For that reason, there appear to be few more cases in which an event with less expected return is placed before one with higher expected return. For that matter, 4-S: AKQJ if QJ fp or 9p (5♡-J♡-Q♡-K♣-A♢) is placed before 2-RF: QJ (2♣-5♡-7♠-J♢-Q♢), 2-S: KJ if JT fp (2♠-5♣-T♠-J♠-K♢) is placed before 2-RF: JT (2♢-T♣-J♣-K♢-A♡), 2-S: AQ if JT

fp (2♣-T♣-J♣-Q♠-A♢) is placed before 2-RF: QT (2♢-5♣-T♠-Q♠-K♢), and 1-RF: K if KT fp and 9sp (2♡-8♣-9♠-T♡-K♡) before 2-RF: KT (2♠-5♣-7♠-T♡-K♡).

Table 3.2: Ethier's Optimal Strategy – Jacks or Better

rank	description	rank	description
1	5-RF, 5-SF, 5-4K, or 5-FH	17	2-RF: AH, KH
2	3-3K	18	3-SF: $s+h=2$, no sp
3	4-2P or 4-RF	19	4-S: AHHT or KQJ9
4	5-F or 5-S	20	3-SF: $s+h=2$
5	4-SF	21	3-S: KQJ
6	2-HP	22	2-S: QJ
7	4-F: $AHTx$+K, Q, J, or T	23	2-S: KJ if JT fp
8	3-RF	24	2-RF: JT
9	4-F	25	2-S: KH
10	4-S: KQJT	26	2-S: AQ if QT fp
11	2-LP	27	2-RF: QT
12	4-S: 5432 – QJT9	28	2-S: AH
13	3-SF: $s+h\geq 3$	29	1-RF: K if KT fp and 9sp
14	4-S: AKQJ if QJ fp or 9p	30	2-RF: KT
15	2-RF: QJ	31	1-RF: A, K, Q, or J
16	4-S: AKQJ	32	3-SF: $s+h=1$

In the mid-1990s, exact optimal strategies for Jacks or Better were attempted by Paymar, Godon, and Chrome, but their results were inaccurate (Ethier, 2010). According to Ethier (2010), it is only relatively recently that a hand-rank table has been constructed and published for Jacks or Better that reproduces the optimal strategy. His hand-rank table coincides with the one of Bob Dancer and Liam Daily (2004). Our first task was to verify this table and we do this by producing a revised table fleshing out different types of discards in Table 3.3.

There seem to be disarrangements of some ranks that are simply misleading even though they do not conflict with the optimal strategy. The reason why his hand-rank table can yet maintain the correct optimal strategy is due to the mutual exclusiveness between misplaced ranks. Assume that rank A and B are mutually exclusive ranks (meaning that they cannot occur at the same time) and A is higher rank than B. Swapping the order of these two ranks will not conflict the optimal strategy because none of the events in B consider events in A due to the mutual exclusiveness. Shortly after considering all the events in B, all the events in A will be played under optimal play. Nonetheless, it seems to be more intuitive to order each mutually exclusive event with the respect to expected return. Consider 4-RF (5♡-T♠-Q♠-K♠-A♠, say) which definitely needs to be positioned before 5-FH (5♠-5♡-Q♡-Q♠-Q♢). The highest form of 4-RF is the outside straight such as KQJT (5♢-T♠-J♠-Q♠-K♠) where

Table 3.3: Optimal Strategy Hand-rank Table for Jacks or Better

rank	description	rank	description
1	5-RF	20	2-RF: *QJ*
2	5-SF	21	4-S: AKQJ
3	5-4K	22	2-RF: *KH, AH*
4	4-RF	23	3-SF: $s + h = 2$, no sp
5	5-FH	24	4-S: AHHT or KQJ9
6	5-F	25	3-SF: $s + h = 2$
7	3-3K	26	3-S: KQJ
8	5-S	27	2-S: QJ
9	4-SF	28	2-S: KJ if *JT* fp
10	4-2P	29	2-RF: *JT*
11	2-HP	30	2-S: KH
12	4-F: *AHTx* + K, Q, J, or T	31	2-S: AQ if *QT* fp
13	3-RF	32	2-RF: QT
14	4-F	33	2-S: AH
15	4-S: KQJT	34	1-RF: K if KT fp and 9sp
16	2-LP	35	2-RF: *KT*
17	4-S: 5432 – QJT9	36	1-RF: A, K, Q, or J
18	3-SF: $s + h \geq 3$	37	3-SF: $s + h = 1$
19	4-S: AK*QJ* if *QJ* fp or 9p		

Note: Hand-rank table at Jacks or Better guides players to choose the best strategy to play in any given initial hand, providing which cards to hold or discard. If none of the ranks applies to the player's hand, then the player should discard all five cards. Abbreviations that are used in this tables are: RF = royal flush, SF = straight flush, 4K = four of a kind, FH = full house, F = flush, S = straight, 3K = three of a kind, 2P = two pair, HP = high pair (jacks or better), LP = low pair (pairs that are not high pair), and the numbers that come before these abbreviations refer to the number of cards in an initial hand that have the potential to become these ranks. n-RF, n-SF, n-F, n-S refer to the n number of cards that have the potential to become RF, SF, F, S. But, if n is equal to the number of cards needed to make up a particular rank, it means that the potential is already realized. 5-RF, 5-SF, 5-FH, 5-F, 5-S, 4-4K, 3-3K, 4-2P, 2-HP, and 2-LP are the hands whose potential is already realized. A, K, Q, J, T denote ace, king, queen, jack, and ten respectively. High card is any of A, K, Q, or J. If multiple cards are italicized, it means that they are of the same suit. s is the number of straight that can be made from the player's hand regardless of the suit, and h denotes the number of high cards in the hand that the player decides to hold in accordance with the strategy. fp, sp, and 9sp stand for flush penalty, straight penalty, and 9 straight penalty card. New types of penalty are introduced. hp represents high card penalty card and shp represents penalty card that is both sp and hp. $n$sp, $n$hp, $n$shp refer to the number of specified penalty cards.

the number of possible straights that can be made from this hand is two and the lowest is inside straight AHHT (5♡-T♣-Q♣-K♣-A♣) where the number of possible straights that can be made from the hand is only one. The expected return of 4-RF ranges from 18.36170 to 19.68085, depending on the remaining card, while that of 5-FH is always 9. It appears that Ethier intended to group all the hands whose potential is already realized first just as he grouped 5-RF, 5-SF, 5-4K, or 5-FH in the same rank in his hand-rank table. If then, 5-F (5♡-7♡-9♡-K♡-A♡) and 5-S (5♠-6♢-7♣-8♡-9♠) should have been placed along with 5-FH, and 4-RF should have been pushed back because 4-RF is not one of the paying hands.

Similarly, 5-F, without concurrently being 5-RF, 5-SF, or even 4-RF, needs to move up before 3-3K (5♠-5♢-5♡-8♠-T♡). 5-F (6) has a unique expected return, which means it has an unchanged expected return for any forms of 5-F. Although 3-3K (3.0250) is open for alternative options such as 4-K (7♠-7♢-7♡-7♣-A♠), clearly 5-F has a higher expected return than any form of 3-3K.

As for 4-2P (3♡-3♠-Q♢-Q♠-6♣), it is placed shortly after 4-SF (4♢-5♢-6♢-8♢-T♣). I felt that it was important to point out that it was not entirely right to think that every event of 4-SF is higher than 4-2P for the following reason. 4-SF can be differentiated by various features present in a hand: types of straight, number of high cards, low pair, and high pair. It was observed that expected return mainly shifted by types of straight. Other factors such as holding high cards and pair are also the key factors that influence the expected return; however, the magnitude of change in expected return is not as crucial as types of straight. The expected return of 4-SF (5♣-6♣-7♣-8♣-A♢) that has outside or open-ended straight ranges from 3.53191 to 3.65957 while that of 4-SF (5♣-6♣-7♣-9♣-A♢) that has inside straight ranges from 2.34043 to 2.53191. Interestingly, 4-2P falls right between these two ranges. This means 4-SF that can form outside straight holds higher weight than 4-2P, and in return 4-2P is higher than 4-SF with inside straight. Due to the mutual exclusiveness between these two ranks and the fact that one form of 4-SF is higher than 4-2P, we decided to place 4-SF before 4-2P in the revised hand-rank table (Table 3.3). However, we needed to impart players that 4-SF does not always hold higher expected return than 4-2P.

Under these modifications and optimal strategy, the payout distribution for Jacks or Better is uniquely determined, and Table 3.4 shows contribution of different hands to overall expected return under optimal play.

$$\text{l.c.m}\left\{ \binom{52}{5}\binom{47}{k} : k = 0,\, 1,\, 2,\, 3,\, 4,\, 5 \right\} = \binom{52}{5}\binom{47}{5}5 \tag{3.3}$$

It follows that the mean payout under optimal play is

$$\frac{1,653,526,326,983}{1,661,102,543,100} \approx 0.995439043695 \tag{3.4}$$

where numerator is calculated by multiplying the last column of the Table 3.4 by the corresponding payoff odds.

Table 3.4: Distribution of expected return from a one-unit bet on Jacks or Better. Assumes maximum-coin bet and optimal drawing strategy.

rank	R	number of ways	probability	probability $\times\binom{52}{5}\binom{47}{5}5/12$
royal flush	800	493,512,264	0.0000247582680376	41,126,022
straight flush	50	2,178,883,296	0.0001093090903715	181,573,608
four of a kind	25	47,093,167,764	0.0023625456858769	3,924,430,647
full house	9	229,475,482,596	0.0115122073362865	19,122,956,883
flush	6	219,554,786,160	0.0110145109680315	18,296,232,180
straight	4	223,837,565,784	0.0112293672413438	18,653,130,482
three of a kind	3	1,484,003,070,324	0.0744486985711364	123,666,922,527
two pair	2	2,576,946,164,148	0.1292789024801780	214,745,513,679
jacks or better	1	4,277,372,890,968	0.2145850311256440	356,447,740,914
others	0	10,872,274,993,896	0.5454346692330940	906,022,916,158
total		19,933,230,517,200	1	1,661,102,543,100

3.6 3-SF

It is noteworthy to single out 3-SF because it appears in the hand-table quite frequently with a variety of restrictions and the three-card straight flush, 3-SF may be the most interesting and delicate rank. Within 3-SF, there may be zero, one, or two inside cards and zero, one, two high cards. Having three high cards has the potential for 3-RF or sometimes even 4-RF with a T so it should not be considered in 3-SF. There are $9 - 1 = 8$ different combinations of insides and high cards, where 1 comes from the case where there are no inside and two high cards (QJT is the only case) whose potential is realized as 3-RF rather than 3-SF. In an attempt to simplify different types of 3-SF within the same strategies, Dancer (2004) introduced a convention to account for all cases. Few years later, Ethier reproduced Dancer's convention in positive integer which made easier for players to comprehend and shortened execution duration. I will explain further how Ethier converted Dancer's convention into better representation.

It is relatively easy to see that the outside straight (having no inside) is the strongest form of straight, and double-inside straight (having two insides) is the weakest form of straight. Even from this remark, one can hypothesize that number of inside card diminishes the value of a 3-SF with regards to expected return. It becomes much apparent once expectations of different types of 3-SF are calculated. Disregarding the penalty cards, 3-SF with two insides ($s = 1$) is worth less than with one inside ($s = 2$), which is worth less than one with no inside at all ($s = 3$). On the other hand, every high card added to 3-SF increases a chance of hitting a high pair such that 3-SF has more worth. Dancer and Liam (2004) discovered that "the negative value caused by an extra inside card can fairly compensate the positive value brought on by an extra high card." In a sense, one extra high card can be "traded off" for one extra inside. Hence, they classified each type of 3-SF as a number by subtracting the number of insides from number of high cards. This resulted in one of +1, +0, -1 or -2. Ethier (2010) introduced a different notation to avoid writing +1, +0, -1 and -2 at the end of 3-SF. He used s and h to denote the number of straights that can be made from 3-SF and the number of high cards within 3-SF. So, when $s = 3$, which is the maximum number straights that can be made from 3-SF, there is no inside within the straight. Likewise, $s = 1$ implies a straight with two insides (double-inside). For an obvious reason, s cannot be zero because if a straight cannot be formed, it should have not been considered as a straight flush in the first place. With the similar idea, the maximum number of h is two (more than two becomes 3-RF), and the minimum number of h is zero. Then, he added both s and h to classify the types of 3-SF, then placed them in a descending order. The highest type of 3-SF ($s + h \geq 3$) is equivalent to SF3 +1 and SF3 +0 in Dancer's winner's guide. There are $s = 2$ and $h = 2$ (4♢-5♡-9♣-J♣-Q♣), $s = 2$ and $h = 1$ (4♣-5♡-9♡-10♡-Q♡), $s = 3$ and $h = 1$ (4♣-5♡-9♠-10♠-J♠), $s = 1$ and $h = 2$ (4♣-5♡-9♢-J♢-K♢) whose sum is at least three. Using Dancer's convention that we subtract the number of insides from the number of high cards to differentiate each 3-SF, the four possibilities listed are equivalent to SF3 +1, SF3 +1, SF3 +0, SF3 +0 respectively. In the same manner, we can observe that Dancer's SF3 -1 and SF -2 are equivalent to 3-SF: $s + h = 2$ and 3-SF: $s + h = 1$ in Ethier's hand-rank table. These different types of 3-SF need to be separated in the table to account for ranks placed between them.

It is also important to analyze 3-SF: $s + h = 2$ with the restriction that allows straight penalty cards or does not allow them. To start with, this specified 3-SF can be formed in two ways. There is a case when both s and h are one, and there is another case when s is two and h is zero. Clearly, the reason for not being able to have two high cards ($h = 2$, $s = 0$) is self-explanatory. These two ranks need to be set apart because there exists a rank (4-S: AHHT or KQJ9) whose expected return falls right between them. Therefore, keeping those two 3-SFs together as one rank would violate the optimal strategy because 3-SF: $s + h = 2$, no sp is preferable over 4-S, in return 4-S is preferable over 3-SF: $s + h = 2$ with sp. For instance, the optimal strategy for a hand like 8♡-9♡-J♣-Q♡-K♠ (falls under 3-SF: $s + h = 2$ with J♣ as a sp) is 4-S, not 3-SF. Therefore, when 3-SF: $s + h = 2$, considering the presence of straight penalty card is important to infer a correct strategy.

Chapter 4

Double Bonus

4.1 Introduction

Players should tolerate much greater proportion of hands and complexities of optimal strategy in Double Bonus. There are many more hand options to consider that are not considered in Jacks or Better, including 3-F, non-consecutive 4-S with certain restrictions, 3-S, and more descriptive 3-RF. Regardless of the complexity of the hand-rank table, the lucrative payoff table itself clearly shows why Double Bonus can be a competitive rival to the popularity of Jacks or Better. To be more specific, Double Bonus is actually one of the rare games whose expected payout exceeds 100 percent under optimal play. As appealing as this game can be, it is known to be an "ugly game" because if the higher paying quads (four of a kind) fail to appear as often as statistically calculated, one should expect costly losing streaks (Dancer & Daily, 2005). However, let us focus on the statistical return that is theoretically promising in the long run.

Double Bonus differs from other video poker games because some casinos allow players the option of either taking their money or doubling the payout. If a player chooses to take the original payout, the hand will end and the player will take the money. But, if a player chooses to double the payout after winning his hand, a fresh set of five cards will be given; one face-up and the rest of the cards face-down. If the card that a player selects out of those four face-down cards is higher than the original face-up card, the payout is doubled; otherwise, the player loses all his payout. One can keep doubling the payout as Double Bonus grants the player the same option of either taking the money or doubling the payout at any time. However, in this paper, the option of doubling the payout is neglected. Here in this game, just like Jacks or Better, it is assumed that the maximum unit is bet because if one bets less than five units, the return of royal flush is only 250 rather than 800 (Dancer

Table 4.1: The full-pay Double Bonus payoff odds, assuming a maximum coin bet, and pre-draw frequencies.

rank	payoff odds	number of ways
royal flush	800 for 1	4
straight flush	50 for 1	36
four aces	160 for 1	48
four 2s, 3s or 4s	80 for 1	144
four of a kind (others)	50 for 1	432
full house	10 for 1	3,744
flush	7 for 1	5,108
straight	5 for 1	10,200
three of a kind	3 for 1	54,912
two pair	1 for 1	123,552
pair of jacks or better	1 for 1	337,920
others	0 for 1	2,062,860
total		2,598,960

& Daily, 2005). Therefore, players should play at the maximum bet at all times to take the full advantage of high payouts such as royal flush and quads.

There are various types of Double Bonus with different payoff schedules just like Jacks or Better. 10/7 Double Bonus (10 denotes the payout for FH and 7 for F) and 10/7/80 Double Bonus (80 denotes the payout for SF) are the only types worth analyzing because other games have slightly different strategies and their returns are significantly less than the other two, so players should not consider them. Statistically speaking, 10/7 and 10/7/80 both bring higher than 100 percent expected returns and clearly higher than the mean expected return in Jacks or Better 9/6. Since the open question on Double Bonus in Ethier's book specifically provides a payoff table for 10/7 Double Bonus and does not mention 10/7/80 Double Bonus, only the former will be discussed. But, it is not difficult to make the modifications to produce a hand-rank table for 10/7/80 Double Bonus if one already has a program that calculates the overall expected return of Double Bonus 10/7. In fact, the mean expected return of 10/7/80 (100.52%) is much higher than that for 10/7 (100.17%). However, players should understand that high volatility in 10/7/80 or even 10/7 (less than 10/7/80) tends to overwhelm the players' budget in the short term even though this game offers positive theoretical return. Players are doomed to lose their bet because they are more likely to experience bankruptcy in the short run. Hence, even though Double Bonus may seem more attractive than Jacks or Better, players should foresee a feasible "pitfall" that might lead to either huge winning or losing streaks (Dancer & Daily, 2005).

4.2 Playing within the Budget

The reason that casinos can advertise "over 100%" video poker games and still make money is due to players' ineptitude and ignorance of the optimal strategy (Dancer & Daily, 2005). This is the reason why casinos can afford to be generous on quads and other low paying hands such as FH and F which pay one unit higher than Jacks or Better. 4-2P is the only hand whose payoff is reduced. So, we were interested in understanding the degree of gain and the loss in these mentioned hands. Under optimal play, approximately 12.47% of hands results in two pair (refer to Table 4.2). Since its payoff is reduced from two to one, the overall expected return is diminished by 0.1247 compared to Jacks or Better. Similarly, as the payoff for flush is increased by one unit, the probability for flush under optimal play shifts from 1.15% to 1.50%. The mean expected payout is increased by an additional 0.35%. In the same way, as the payoff for full house is also increased by one unit, its probability under optimal play is adjusted from 1.15% to 1.12%. Although the frequency of full house is decreased by a slight, the fact that the payoff is increased by one unit brings up a surplus in the mean expected return. In addition to flush and full house, quads is the critical key factor that changes the theoretical return in an optimistic way and what allows players to have an edge over the casino in the long run. In Double Bonus, quads or four of a kind, being a comparatively infrequent event, is nonetheless accountable for a significant portion of the total return under optimal play (15.38% = 0.1541149/1.00175224). In comparison, quads in Jacks or Better is only responsible for 5.93% (0.0590636/0.9954390) of the total return. This figure implies that it is difficult for players to have positive overall return without hitting quads in Double Bonus.

Despite the positive expected payout, Bob Dancer's simulations (2005) suggested that if one bets large amount that is unsuitable for one's budget, one would not survive financially before experiencing the taste of high paying hands because there will be big swings in both ways in the short term. Bob Dancer's simulations warned the volatility of the game that players should be fully aware of and also explained the reason why players should play within their budget. He ran 100,000 samples of 5,000 hands (average number of hands dealt in 8 hours) played under optimal strategy. 10% of these samples resulted in a loss of $1,995 or greater when each hand was played at $1 denomination. The lowest peak of the loss in these 10% was at least $2,330. In 5% of samples, the minimum loss was $2,425 while the lowest peak of the loss was $3,365. At the 1% level of simulations, the minimum loss was $3,175 with the lowest peak of $3,365. This means despite playing under optimal strategy, a player should expect enormous losses on an unlucky day. It further implies that due to the volatile frequency of quads, the game in the short term tends to be overwhelmed especially if one bets more than one's budget.

Table 4.2: The distribution of expected return from a one unit bet on Double Bonus, assuming maximum-coin bet and optimal drawing strategy.

rank	R	number of ways	probability	probability $\times\binom{52}{5}\binom{47}{5}5/12$
royal flush	800	414,860,472	0.0000208125056118	34,571,706
straight flush	50	2,254,539,360	0.0001131045646642	187,878,280
four aces	160	3,962,537,892	0.0001987905517162	330,211,491
four 2s, 3s, or 4s	80	10,446,264,684	0.0005240628043200	870,522,057
four of a kind (others)	50	32,045,992,176	0.0016076667627131	2,670,499,348
full house	10	223,050,658,608	0.0111898900890919	18,587,554,884
flush	7	298,068,519,408	0.0149533473337803	24,839,043,284
straight	5	299,385,345,432	0.0150194091807480	24,948,778,786
three of a kind	3	1,439,168,246,364	0.0721994483093029	119,930,687,197
two pair	1	2,484,844,034,580	0.1246583704751660	207,070,336,215
high pair	1	3,834,735,881,328	0.1923790465383460	319,561,323,444
others	0	11,304,853,636,896	0.5671360508845400	942,071,136,408
total		19,933,230,517,200	1	1,661,102,543,100

Nonetheless, the mean payout under optimal play has a positive return of

$$\frac{1,663,968,316,328}{1,661,102,543,100} \approx 1.001725224 \tag{4.1}$$

on a one-unit bet. Thus, optimal Double Bonus players have a slight advantage (about 0.17 percent) over the house. The hand-rank table (Table 4.3) giving the strategy that achieves such a return is as follows.

Table 4.3: Optimal Strategy Hand-rank Table for Double Bonus

rank	description	rank	description
1	5-RF	24	2-RF: QJ
2	5-4K	25	3-F: HHx
3	5-SF	26	2-RF: KH
4	4-RF	27	3-SF: $s + h = 2$, no sp
5	3-3K with aces	28	2-RF: AH
6	5-FH	29	3-SF: $s + h = 2$
7	5-F	30	4-S with 2H
8	3-3K	31	3-S: KQJ
9	5-S	32	3-S: QJT
10	4-SF	33	4-S with 1H
11	4-2P	34	2-RF: JT
12	2-HP with aces	35	2-S: QJ
13	3-RF: QJT	36	3-SF: $s + h = 1$
14	2-HP	37	3-F: Hxx
15	4-F	38	2-RF: QT
16	3-RF	39	3-S: AHH if 9p
17	4-S: 5432 – KQJT	40	2-S: AH, no sp
18	2-LP with 2s, 3s, and 4s	41	2-S: KH
19	3-SF: $s + h > 3$	42	1-RF: A
20	2-LP with others	43	2-RF: KT
21	3-SF: $s + h = 3$	44	1-RF: J, Q, or K
22	4-S: AKQJ	45	4-S with 0H
23	4-S: AHHT or KQJ9	46	3-F

Note: Hand-rank table at Double Bonus guides players to choose the best strategy to play in any given initial hand, providing which cards to hold and discard. If none of the ranks applies to the player's hand, then the player should discard all five cards. Abbreviations that are used in this tables are: RF = royal flush, SF = straight flush, 4K = four of a kind, FH = full house, F = flush, S = straight, 3K = three of a kind, 2P = two pair, HP = high pair (jacks or better), LP = low pair (pairs that are not high pair), and the numbers that come before these abbreviations refer to the number of cards in an initial hand that have the potential to become these ranks. n-RF, n-SF, n-F, n-S refer to the n number of cards that have the potential to become RF, SF, F, S. But, if n is equal to the number of cards needed to make up a particular rank, it means that the potential is already realized. 5-RF, 5-SF, 5-FH, 5-F, 5-S, 4-4K, 3-3K, 4-2P, 2-HP, and 2-LP are the hands whose potential is already realized. A, K, Q, J, T denote ace, king, queen, jack, and ten respectively. High card is any of A, K, Q, J. If a multiple cards italicized, it means that they are of the same suit. s is the number of straight that can be made from the player's hand regardless the suit, and h denotes the number of high cards in the hand that the player decides to hold in accordance with the hand-rank table. fp, sp, and 9sp stand for flush penalty, straight penalty, and 9 straight penalty card. New types of penalty are introduced. hp represents high card penalty card and shp represents penalty card that is both sp and hp. $n$sp, $n$hp, $n$shp refer to the number of specified penalty cards.

4.3 New and Modified Ranks Not in Jacks or Better

The hand-rank table for Double Bonus (Table 4.3) presents a slightly different optimal strategy than Jacks or Better. Changes in payoff odds in this game required befitting order of each rank under optimal strategy. Therefore, new ranks are introduced that have not appeared in Jacks or Better to exhaust the range of all the holdings. These are derived and modified from the Jacks or Better hand-rank table to satisfy the optimal strategy for Double Bonus.

The first apparent adjustment in the hand-rank table for Double Bonus is that 3-3K (A♣-A♡-A♠-4♠-7♡), holding a three of kind, is sorted into two parts: one with aces and one without. The reason is clear: 3-3K with aces holds outstandingly more significance that leads to achieving quads whose payout is more lucrative than one for other denominations. The payoff is at least doubled. So, even when 5-FH (A♣-A♡-A♠-J♠-J♡) is formed with three aces, which is already a paying hand, the optimal strategy suggests that a player to discard a pair in order to allow for the possibility of quads with aces. It sounds like poor play to give up a hand that already promises to pay 10 units (payoff for FH), which now one might eventually have to be content with receiving 3 units (payoff for 3K) by giving up a pair. However, the theoretical expectation assures that giving up 5-FH for possible 4-K with aces (10.11471) will receive greater share in the long run if a player decides to hold three aces rather than 5-FH (10). 3-3K with 2s, 3s, 4s and 3-3K with other denominations need to be distinctively separated due to their different payoff odds, but they are put in the same rank because there is no rank positioned between three 2s – 4s and three 5s – Ts.

Within 3-RF and 4-F, there are more options to consider. The combinations of high cards and straights that can be made from the hand are sensitive in 3-RF, as well as in 3-SF in Double Bonus because their returns and strategies are very close to the other ranks. 3-RF: QJT ($s = 3$) is the strongest form of 3-RF, and as a result it has the highest return. Then, there comes special forms of 4-F: $KQJx$ (3♢-5♠-J♠-Q♠-K♠) and $AHTx$ + K, Q, J, or T (4♠-T♠-J♡-K♠-A♠) before the rest of other types of 3-RF. Then, 3-RF: KQJ, 2-HP, 4-F, and 3-RF (others) are placed in that order with respect to their returns. Equivalently, we can simplify the order of these ranks as follows: 3-RF: QJT, 4-F (any types), 3-RF: KQJ, 2-HP, and 3-RF (others). These findings were tested by simply taking all the $\binom{5}{3}$ combinations out of all five royal cards along with combinations of the remaining two cards. Then, I recorded the holdings along with the optimal strategy of each combination, and I ranked them with the respect to their conditional expected returns. 3-RF, in our opinion, can be easily mistreated since different types of 3-RF do not affect the optimal strategy in Jacks or Better.

3-SF is also an interesting case in Double Bonus. As I attempted to use the same organization of 3-SF applied in Jacks or Better, I found that there is a minor change needed in order to satisfy the optimal strategy. There has to be a distinction between 3-SF whose $s + h$ is greater than 3 and $s + h$ is equal to 3 because 2-LP with 5s – Ts is preferred over 3-SF when the sum is three or less. This result was observed when I initially tried to plug in different types of hands within $s + h \geq 3$ as used in Jacks or Better. I came to know that 3-SF: $s + h > 3$ (4♡-5♢-9♠-T♠-J♠) is better than 2-LP (4♣-8♠-8♢-9♢-T♢) with 5s

– Ts (not 2s – 4s), in return is superior than 3-SF: $s + h = 3$ (4♣-5♠-8♢-9♢-T♢). This can be shown when 3-SF: $s + h > 3$ occurs with 2-LP (5s – Ts). The optimal strategy for a hand that matches that particular description (3-SF: $s + h > 3$) is to keep the 3-SF cards, and 3-SF: $s + h = 3$ is neglected by 2-LP (5s – Ts) under optimal play. The same procedure was executed to place the rest of 3-SFs according to their returns.

One of the new ranks that are introduced in Double Bonus is 3-F. Payoffs for royal flush and straight flush remained unchanged but the one for flush is increased by one. This amplifies the overall value of flush such that holding three cards in the same suit has considerable potential. Based on a similar idea of 4-F: $AHTx$ + K, Q, J, or T in Jacks or Better, I came to a realization that the number of high cards in 3-F is significant factor that regulates the expected return. By its set up, a hand cannot have more than two high cards or it would form 3-RF or greater. As the hand-rank table progresses, 3-F with one high card (2♣-3♡-6♠-8♡-K♡) and 3-F with no high card (2♣-3♡-6♠-8♡-T♡) are established accordingly. Proceeding to the bottom of the hand-rank table, there are numerous ranks that are placed between different types of 3-F. This means that combining them into just one 3-F will conflict with the optimal strategy.

For the same reason as 3-F, a straight with one inside (2♣-8♡-9♠-J♡-Q♣) is observed significant enough to be considered in the hand-rank table. I learned that the number of high cards within 4-S is the key that classifies different levels of 4-S and affects a value of return and all four cards need not to be in the same suit. A set of cards may be suited as long as it does not match any description in higher ranks such as royal flush, straight flush, or even pairs. Penalty cards cannot physically be present along with 4-S because it conforms to a straight. Although hp can be present (except for 4-S with 0H), it does not influence the expectation. Nonetheless, having to remember the correct placement of different levels of 4-S stalls players from using the optimal strategy while playing Double Bonus. The level of complexity in different forms of 4-S provides casinos with some assurance that Double Bonus is indeed a profitable game for them despite expected return.

Chapter 5

Joker Wild

5.1 General Introduction

Joker Wild is a variant of Jacks or Better played with 52 standard cards supplemented with joker card. A joker is a wild card meaning that it can be considered as any of original 52 cards in the deck, even ones that are already appeared and possibly discarded. This means with a joker present, different hands may be possible. Thus, the presence of a joker adds another level of excitement in anticipating more hands will fall into high paying hands. But, among all possible ways that lead to positive payout, the joker should be used in a way that can bring out the highest payout available. Due to the fact that a joker allows too many hands in a favor of players, casinos have adjusted the requirement of high pair from jacks or better to kings or better. Regardless, the payoff schedules seem to be modified so that players have a little more house advantage over casinos. Anticipating what we will show below, Joker Wild video poker has a 100.65% return under optimal play which is even higher than the one for Double Bonus. As glamorous and thrilling as it sounds, not to mention complexity and sophistication of each rank, the number of ranks that players have to digest is almost doubled because strategy tables need to consider both the presence and absence of a joker. The optimal strategy for Joker Wild is quite different because a joker that can form various hands and that only king or ace is allowed as a high card. Table 5.1 presents the pre-draw frequencies of Joker Wild.

Table 5.1: Confirming the pre-draw frequencies Joker Wild

rank	payoff	number of ways without a joker	number of ways with a joker
royal flush (naural)	800 for 1	$\binom{5}{5}\binom{4}{1} = 4$	0
five of a kind	200 for 1	0	$\binom{13}{1}\binom{4}{4} = 13$
royal flush (joker)	100 for 1	0	$\binom{5}{4}\binom{4}{1} = 20$
straight flush	50 for 1	$[\binom{10}{1} - 1]\binom{4}{1} = 36$	$[\binom{5}{4}\binom{9}{1} - 9]\binom{4}{1} = 144$
four of a kind	20 for 1	$\binom{13}{11,1,0,0,1}[\binom{4}{1}\binom{4}{4}] = 624$	$\binom{13}{11,1,0,1,0}[\binom{4}{3}\binom{4}{1}] = 2{,}496$
full house	7 for 1	$\binom{13}{11,0,1,1,0}[\binom{4}{2}\binom{4}{3}] = 3{,}774$	$\binom{13}{11,0,2,0,0}[\binom{4}{2}^2] = 2{,}808$
flush	5 for 1	$[\binom{13}{5} - \binom{10}{1}]\binom{4}{1} = 5{,}108$	$\binom{13}{4}\binom{4}{1} - 164 = 2{,}696$
straight	3 for 1	$\binom{10}{1}[\binom{4}{1}^5 - \binom{4}{1}] = 10{,}200$	$[\binom{10}{1}\binom{5}{4} - \binom{9}{1}][\binom{4}{1}^4 - \binom{4}{1}]= 10{,}332$
three of a kind	2 for 1	$\binom{13}{10,2,0,1,0}[\binom{4}{1}^2\binom{4}{3}] = 54{,}912$	$\binom{13}{10,1,2,0,0}[\binom{4}{2}\binom{4}{1}^2]= 82{,}368$
two pair	1 for 1	$\binom{13}{10,1,2,0,0}[\binom{4}{1}\binom{4}{2}^2] = 123{,}552$	0
aces or kings	1 for 1	$\frac{2\binom{13}{9,3,1,0,0}[\binom{4}{1}^3\binom{4}{2}]}{13}$ =168,960	$[\binom{13}{4} - \binom{11}{4} - 12][\binom{4}{1}^4 - \binom{4}{1}]= 93{,}996$
other	0 for 1	$\binom{52}{5} - 367,140 = 2,231,820$	$\binom{52}{4} - 194,873 = 75,852$
total		$\binom{52}{5} = 2{,}598{,}960$	$\binom{52}{4} = 270,725$

5.2 Equivalence Classes for Joker Wild

The addition requires another look at the equivalence relation defined for Jacks or Better. If a joker is absent in the initial hand, then the equivalence classes are exactly the same as in Jacks or Better or Double Bonus. But, the list of equivalence classes of initial player's hand, together with the size of each equivalence classes is definitely different if a joker is present (refer to Table 5.2). By direct enumeration, there are exactly

$$\binom{13}{4}15 + \binom{13}{10,3,1,0,0}6 + \binom{13}{11,0,2,0,0}3 + \binom{13}{11,1,0,1,0}2 + \binom{13}{12,0,0,0,1}1 = 16,432 \tag{5.1}$$

equivalence classes together with 134,459 equivalence classes in cases of absence of joker (a sum of 150,891 equivalence classes in total). As a check, we can compute the total number of hands by summing the sizes of the equivalence classes:

$$\begin{aligned}
&\binom{13}{4}(1 \cdot 4 + 7 \cdot 12 + 7 \cdot 24) + \binom{13}{10,2,1,0,0}(4 \cdot 12 + 2 \cdot 24) + \binom{13}{11,0,2,0,0}(2 \cdot 6 + 1 \cdot 24) \\
&+ \binom{13}{11,0,2,0,0}(1 \cdot 12 + 1 \cdot 4) + \binom{13}{12,0,0,0,1}1 + \binom{13}{5}(1 \cdot 4 + 15 \cdot 12 + 35 \cdot 24) \\
&+ \binom{13}{1,3,9}(8 \cdot 12 + 12 \cdot 24) + \binom{13}{1,2,10}(1 \cdot 4 + 7 \cdot 12 + 5 \cdot 24) + \binom{13}{1,1,11}(1 \cdot 4 + 2 \cdot 12) \\
&= 270,725 \text{ (when joker is present) } + 2,598,960\text{(when joker is not present)} \\
&= 2,869,685 \\
&= \binom{53}{5} \qquad (5.2)
\end{aligned}$$

Our computer program provides the strategy for each of the 150,891 equivalence classes. The equivalence classes are divided into two main parts such that the program considers 134,459 equivalences that do not have a joker in a hand and 16,432 equivalence classes that include a joker in a hand. The hardest part of modification in constructing a hand-rank table (Table 5.3) is to figure out a way to represent a joker in the program. Whenever denomination of a card needs to be expressed, a joker is assigned 1 as its denomination and 0 as its suit while the rest of the cards are expressed from 2 to 14 as their denominations with the corresponding suits from 1 to 4. Then, we execute the program to find the hand that matches the highest feasible qualifying hand and store the result. To do so, I needed to manually list all the combinations of all the paying hands with a joker. For example, if a joker is not present in a hand, 10-11-12-13-14 with (1,1,1,1,1) is the only hand qualifying for 5-RF. If a joker is present in a hand, any of 1-10-11-12-13, 1-10-11-12-14, 1-10-11-13-14, 1-11-12-13-14 with (0,1,1,1,1) is qualified for 5-RF, where 10, 11, 12, 13, 14 are T, J, Q, K, A respectively. The program had to be constructed so that it is able to identify all four cases of 5-RF. In a similar manner as the example shown above, I needed to manually catalog different combinations of hands for all paying ranks so that the program could recognize them.

Table 5.2: List of equivalence classes with a joker and size of each equivalence class (asterisk denotes a joker).

five distinct denominations (*, a, b, c, d): $\binom{13}{4} = 715$ ways (include hands ranked pair, straight, flush, straight flush, royal flush)							
(*, 1, 1, 1, 1)	4	(*, 1, 1, 2, 3)	24	(*, 1, 2, 2, 1)	12	(*, 1, 2, 3, 2)	24
(*, 1, 1, 1, 2)	12	(*, 1, 2, 1, 1)	12	(*, 1, 2, 2, 2)	12	(*, 1, 2, 3, 3)	24
(*, 1, 1, 2, 1)	12	(*, 1, 2, 1, 2)	12	(*, 1, 2, 2, 3)	24	(*, 1, 2, 3, 4)	24
(*, 1, 1, 2, 2)	12	(*, 1, 2, 1, 3)	24	(*, 1, 2, 3, 1)	24		
three of a kind (*, a, a, b, c): $\binom{13}{10,2,1} = 858$ ways							
(*, 1, 2, 1, 1)	12	(*, 1, 2, 1, 3)	24	(*, 1, 2, 3, 3)	12	(*, 1, 2, 3, 4)	12
(*, 1, 2, 1, 2)	12	(*, 1, 2, 3, 1)	24				
full house (*, a, a, b, b): $\binom{13}{11,2} = 78$ ways							
(*, 1, 2, 1, 2)	6	(*, 1, 2, 1, 3)	24	(*, 1, 2, 3, 4)	6		
four of a kind (*, a, a, a, b): $\binom{13}{1,1,11} = 156$ ways							
(*, 1, 2, 3, 1)	12	(*, 1, 2, 3, 4)	4				
five of a kind (*, a, a, a, a) $= \binom{13}{12,1} = 13$ ways							
(*, 1, 2, 3, 4)	4						

5.3 Order of Ranks in the Hand-rank Table

Let us consider the first part of the hand-rank table in the absence of a joker. There are no surprises or noticeable changes for holding three or more cards. But, once the optimal strategy is narrowed down to holding two or less cards, it can be observed that various holdings are extremely sensitive to fp, sp, 9p, and even combinations of those penalty cards. Holding AK in the same suit (2♠-8♠-9♡-K♣-A♣) is the first occurrence of holding two or fewer cards. AK seems to be appropriate for the reason that not only they are both high cards but also they can open up wide selections of paying hands except for five of a kind. Now, we wish to consider expected return and placement of unsuited AK in hand-rank table. 2-S: AK if one of (KQ, KJ, KT) fp and 9p (7♡-9♢-Q♡-K♡-A♣) is placed before 2-RF: KQ, KJ, KT (2♣-5♢-7♠-Q♡-K♡). Although generally speaking, 2-RF has higher expected return than holding AK, having an ace as a shp (straight and high card penalty card) along with 9p (double-inside straight penalty card) decreases the value of holding 2-RF so that it is preferable for players to hold AK over 2-RF, even though the difference in expectation is very small between these two strategies ($E[R_{AK}] - E[R_{KQ,KJ,KT}] = \frac{7{,}793-7{,}789}{\binom{48}{3}} = \frac{4}{\binom{48}{3}}$ in 7♡-9♢-Q♡-K♡-A♣), Likewise players should hold 2-S: AK if a hand is one of the following AKQJ, AKQT, AKJT and where one of (AQ, AJ, AT) is suited (2♠-J♢-Q♣-K♡-A♣).

Table 5.3: Optimal Strategy Hand-rank Table for Joker Wild

rank	description	rank	description
Without a joker			
1	5-RF	19	3-SF: $s+h=2$
2	5-SF	20	2-RF: AK
3	5-4K	21	4-S: 5432 – QJT9
4	4-RF	22	3-SF: $s+h=1$
5	5-FH	23	2-S: AK if one of (KQ, KJ, KT) fp and 9sp
6	5-F	24	2-RF: KQ, KJ, KT
7	4-SF	25	2-S: AK if one of (AKQJ, AKQT, AKJT)
8	3-3K		if one of (AQ, AJ, AT)
9	4-S	26	1-RF: A: AKQJx, AKQTx, AKJTx if x is 6-9
10	4-2P	27	2-S: AK
11	3-RF: $s+h=3$	28	1-RF: A if one of (AQ, AJ, AT) fp
12	2-HP	29	2-RF: AQ, AJ, AT
13	3-RF: $s+h=2$	30	1-RF: A or K
14	4-F	31	2-RF: JT
15	3-SF: $s+h=3$, sp	32	Discard everything if QT fp with J
16	2-LP		or QT fp with 2sp
17	3-SF: $s+h=3$	33	2-RF: QT
18	4-S: KQJT		
With a joker			
1	5-5K	18	2-HP: K if 4-S with A or if A fp
2	5-RF	19	2-HP: A if AK
3	5-SF	20	3-SF ($s=3$): *46 – 8T* if no sp, no fp
4	5-4K	21	2-HP: A
5	4-RF	22	4-S: KQJ, KQT, KJT
6	5-FH	23	2-HP: K
7	4-SF: $s=3$	24	3-SF ($s=3$): *35 – 9J*
8	5-F	25	4-F: *25* or *9Q*
9	4-SF: $s=1, 2$	26	3-SF ($s=2$): *25x – 9Qx*
10	3-3K	27	4-F
11	5-S	28	4-S: 456 -89T
12	3-RF: KQ, KJ, KT	29	4-S: 345 + J or Q, 9TJ + 2, 3, or 4
13	4-F: HHx or Hxx	30*	2-LP: 6, 7, 8
14	3-RF: With A or JT	31*	2-LP: T
15	3-SF: $s=4$	32*	2-LP: 5, 9
16	3-SF with A or K	33	2-LP: J if J432 or QJ43
17	3-RF: QJ, QT	34	2-LP: 4 if QJ42

Note: Choose the corresponding strategy ranked highest in the table. If a joker is present in an initial hand, players are to hold a joker and a set of card(s) that matches the description of the highest rank available in the table. If none applies, hold only the joker. Note that description of each hand is formed with the joker. Abbreviations used in Jacks or Better or Double Bonus have been also used here with an addition of 5-5K which is a five of a kind with a joker. In Joker Wild, A and K are only considered as high cards because 2-HP is qualified for kings or better. Asterisk means some exceptions apply (See 5.5 – 5.6).

Holding AK over 2-RF gives an additional $\frac{7{,}743-7{,}684}{\binom{48}{3}} = \frac{59}{\binom{48}{3}}$ expected return.

Just before the rest of 2-S: AK, 1-RF: A if AKQJ if there is no straight penalty card for A (6–9) (6♣-J♢-Q♣-K♡-A♠) needs to be positioned for the reason that 1-RF: A if AKQJ if no sp is a subset of event of 2-S: AK. If the order of these ranks were switched, this will conflict with optimal strategy ($E[R_A]-E[R_{AK}] = \frac{87{,}174}{\binom{48}{4}} - \frac{7{,}743}{\binom{48}{3}} = 0.00033533$). Once again, the difference in expected return between two strategies is small. Practically, this means that choosing the weaker strategy does not have much of an effect on the mean payout. Nonetheless, these examples show that our hand-rank table is delicate enough to account for all the ranges of possible hands under right play.

As expected, 1-RF: A or K, as only high cards, behave sensitively with various combinations of penalty cards and lead to unique strategies under different circumstances (rank 23 – 30). But these ranks are straightforward as they are simply ordered in an accordance with optimal strategy and they are supported by representative hands in the Appendix C. I want to point out an event in which players should discard all five initial cards (rank 32) before the end of the hand-rank table if QT is fp with J (2♠-6♣-T♣-J♢-Q♣) or QT is fp with 2sp (2♢-8♠-9♢-T♣-Q♣). It is quite surprising finding that such a hand exists before the end of the hand-rank table.

This rank contains an event in which QT are suited with another card that is free of straight or high penalty and there exists J in the different suit as a straight penalty card. Other equivalent occurrence is when QT are suited with another card that is free of penalty and there exists a combination of any two straight penalty cards such as 8 and 9 or 8 and J. Even though rank 33 suggests that holding QT may be a smart strategy in some cases, list of examples in the table (in Appendix C) help us deduce that presence of sp (especially outside sp) and fp (as specified in rank 32) moderate the value of holding QT that discarding all the cards is the most intelligent tactic.

On average, a joker appears in about one out of every ten hands ($\frac{5}{53} = 0.09434$) and increases the chance of getting a payoff. For the reason that a joker can be used as any card in the deck and uniquely used to achieve the highest paying rank, one can presume that the strategy table with holding the joker will look different from the one without. One outstanding ordering structure identified in this hand-rank table is 3-RF. The highest form of 3-RF with the joker is either of KQ, KJ, KT, having a king as a high card with two possible ways of forming straights. Then, JT whose existence along with the joker AQ, AJ, AT is placed subsequently. Players should be aware that the strategy to hold 3-RF with an ace or JT is no longer applicable if there exists a flush penalty card. If a flush penatly card exists, it conforms to 4-F: HHx and Hxx (rank 13) which is slightly superior over 3-RF (with an ace or JT). Others ranks discussed in the hand-rank table are relatively easy to comprehend but this case shows some of the subtleties.

5.4 Optimal Strategy for Holding a Pair with a Joker

This section may be perhaps the most fascinating to players who want to study each rank more extensively and master the optimal strategy. Just after rounds of ranks with holding three cards, more delicate restrictions are applied to the rest of the ranks especially when holding a pair with the joker is the best decision to bring out the highest return. When players are advised to hold a pair with a joker under optimal play, we have to consider various combinations of penalty cards. Additional explanations and examples will be provided as we suggest a way to decide which is the most valuable card that players should play together with the joker.

There is a lot of work to be done when A and K both occur at the same time because now we have to take various combinations of penalty cards into consideration in order to maintain the optimal strategy. For that reason, the decomposition of ranks that hold A or K is inevitable (rank 18 to 23). However, once the order the optimal strategy for these events is established, it is easy to follow. Holding 2-LP with the joker (rank 30 to 32) also deserves some explanation. If we lay out the cards from 2 to Q, it is fairly easy to recognize that 6, 7, 8 are the only cards with which a hand cannot possibly run a straight with any of high cards (A or K).

$$A \quad 2 \quad 3 \quad 4 \quad \overbrace{5 \quad \underbrace{6 \quad 7 \quad 8} \quad 9} \quad T \quad J \quad Q \quad K \quad A \tag{5.3}$$

By symmetry, 5 and 9 have the potential to draw a straight with a high card (5 with A and 9 with K). On the other hand, T can make a straight with one of the two high cards or possibly both. Although 5, 9, and T form equal number of straight flush, a part of SF that T can make is reserved for RF. For the fact that T benefits lucrative return in royal flush category while 5 or 9 does not, T is generally more valuable than 5 or 9. However, it can be learned through mutiple examples provided in Appendix C (or even in Example (5.8)) that hands with the potential to form a straight with a high card have less return in HP (kings or better) category than the ones without. The reason is because when we consider the value of a hand in HP category, we have to count out the events that we already considered as SF and RF to avoid double counting. But 5 or 9 does not experience this problem so they beat T in this category. Nonetheless, even though T experiences more loss in HP category, gain in RF makes T more valuable than 5 or 9 (some exceptions apply based on different combinations of penalty cards). In addition, there are some exceptional cases in which players are advised to hold 4 or J in the absence of cards from 5 to T. J is put first because when it occurs with 4 simultaneously, J is always the optimal strategy. Understanding the attributes of each group of cards in mind, we can conclude that

$$\{6, 7, 8\} > T > \{5, 9\} > J > 4 \tag{5.4}$$

However, the statement is not entirely true. We have seen that different combinations of straight penalty cards placed upon the optimal holding can slightly affect the expected return. It is extremely complicated to list all the possible holdings of cards along with different combinations of straight penalty cards. Therefore, a way of identifying the value of each straight penalty card around possible holding which works for every case needs to be systemized and ideally ranked.

The straight penalty cards can be categorized into four ways determined by the number of possible straights with the optimal holding. Different types of sp (double-inside, inside, outside, outside1) slightly change the expected value and affect the optimal strategy. Therefore, a straight penalty card is significant enough to be considered as an important factor that determines the optimal strategy. If the optimal card can form one possible straight ($s = 1$) with another card, that card is defined as double-inside sp. If $s = 2$, it is defined as inside sp; if $s = 3$, outside sp; if $s = 4$, outside1 sp. Hypothetically, suppose 8 is the optimal holding for a particular hand, then 4 or Q is designated as double-inside, 5 or J as inside, 6 or T as outside, 7 or 9 as outside1 sp. The higher s is, the higher degree at which straight penalty (sp) card can diminish the expected return. Hence, having 5 as a straight penalty card brings a slightly higher expected return than having 7 as a straight penalty card with an assumption that the optimal strategy remains to be 8.

Let us now assume that we do not know what the optimal strategy is. If so, we do the following. Aside from the joker, we pick the ith card to see how many straight or s it can make with the jth card, where $i \neq j$. Then we add up each s from all the js. We call the summation of all the straight penalty cards corresponding to the selected ith card, s_i, where $i \in \{1, 2, 3, 4\}$ (there is no 5 because we exclude the joker). To give an example, if ★-5♡-7♠-8♢-T♣ is dealt (where ★ means a joker), first point of pivot is 5, and we count the possible straights with 7, which is $s = 2$; with 8, $s = 3$; with T, $s = 0$. The sum of each s corresponding to 5 is five. After that, we repeat this process for the rest of the cards 7, 8, and T ($s_7 = 9, s_8 = 9, s_{10} = 5$). Even though both 5 and T have the same number of possible s that can be made, there is a slightly higher advantage to choosing T over 5 by $\frac{11}{\binom{48}{3}}$ as discussed in (5.4). To better the ranking of cases in which more than one card can have the same count of s, we define a system that determines which card of holding is more valuable than the others under any circumstances.

$$\begin{aligned} &\text{If } \min(s_i) - \min(s_j) \geq 2 \text{ where } i = 6, 7, 8 \text{ and } j = 5, 9, T \text{ then choose } j \text{ over } i \\ &\text{If } \min(s_i) - \min(s_j) < 2 \text{ where } i = 6, 7, 8 \text{ and } j = 5, 9, T \text{ then choose } i \text{ over } j \end{aligned} \tag{5.5}$$

$$\begin{aligned} &\text{If } \min(s_i) > \min(s_j) \text{ where } i = T \text{ and } j = 5, 9 \text{ then choose } j \text{ over } i \\ &\text{If } \min(s_i) \leq \min(s_j) \text{ where } i = T \text{ and } j = 5, 9 \text{ then choose } i \text{ over } j \end{aligned} \tag{5.6}$$

For the future reference, we denote as group one any card from {6, 7, 8}, group two to be T, and group three any card from {5, 9}. If only the cards in the same group are present in a hand, we select the card that has the lowest count of s to be the optimal strategy. For example, there can be hands like ★-2♢-<u>6</u>♣-8♡-J♠, ★-2♢-3♣-4♡-<u>T</u>♠, ★-2♢-<u>5</u>♣-9♡-J♠, where underlined card represents the optimal holding along with the joker due to its lowest count of s within the same group. Certainly they are cases in which there are more than one lowest count, such as ★-2♢-6♣-8♡-Q♠ (6 and 8 are in the same group and have the same number of count), and then either of those cards can be the optimal strategy as they give the exact same expected return. This strategy is then nonuniquely determined by player's perference.

(5.5) and (5.6) were constructed as we considered the possible combinations of straight penalty cards at different positions within and between the groups and determined which holding of cards became the optimal strategy. Generally speaking, cards from group one are higher than cards from the other groups. They are more "valuable" such that even allowing at most one more count of s than 5, 9, T, the desirable choice remains to be group one cards $\{6, 7, 8\}$. From the example that we discussed earlier, when ★-5♢-7♣-8♡-T♠ is the initial hand, 5♢ and T♠ have the same count of s because 5♢ and T♠ are symmetric against 7♣ and 8♡. $E[R_T] - E[R_5] = \frac{11}{\binom{48}{3}}$ implies that the difference in value of two expectations is almost negligible. Placing an extra count of s on T♠ with other restrictions remaining the same (such a hand as ★-3♢-5♣-T♡-J♠) is at its most value if 5♣ is now selected to be the strategy along the joker. Hence, between group two and group three, we hold the card that gives the lowest count of s although generally T has slightly greater value than 5 or 9.

As we have mentioned before, there are exceptionally rare cases in which players are advised to play 4 or J with the joker as their optimal strategy. There are so few that we listed out the cases in the hand-rank table (rank 33 and rank 34). It is fairly easy to notice that all these cases are in the absence of 5 – T. In the events when both 4 and J appear concurrently, giving the same position of sp around them such as ★-3♢-4♣-J♡-Q♠, J is worth more than 4. Contribution to the expected return of holding J comes from RF, while holding 4 cannot make a use of RF as an advantage. However, their difference is so minimal that if $s_J > s_4$, the optimal play is changed to 4 over J. Such a hand that matches description of that particular event is ★-2♢-4♣-J♡-Q♠ ($s_4 = 3, s_J = 4$). This leaves out a case of ★-2♢-3♣-J♡-Q♠ when we attempt to make hypothetical hand with one of 4 and J or both. In this case, we discard every card except for the joker ($E[R_{joker}] - E[R_{joker,J}] = \frac{280{,}720}{\binom{48}{4}} - \frac{24{,}853}{\binom{48}{3}} = 0.0057753$).

A hand calculation is provided to show contribution of each rank to the final expected return and to determine the optimal strategy. If ★-2♢-3♣-8♡-T♠ is dealt, calculation shows that holding the joker and 8♡ is the desirable choice.

$$
\begin{aligned}
E[R_8] \;=\;& \underbrace{200\frac{1}{\binom{48}{3}}}_{\text{5-K}} + \underbrace{0}_{\text{RF}} + \underbrace{50\frac{\left[\binom{5}{1}\binom{4}{1}-4\right]}{\binom{48}{3}}}_{\text{SF}} + \underbrace{3\frac{\binom{9}{1}\left[\binom{4}{1}^3-1\right]+\binom{7}{1}\left[\binom{4}{1}^2\binom{3}{1}-1\right]}{\binom{48}{3}}}_{\text{S}} \\
&+ \underbrace{20\frac{\binom{3}{2}\binom{45}{1}-\binom{9}{1}\binom{4}{3}+\binom{3}{1}}{\binom{48}{3}}}_{\text{4-K}} + \underbrace{7\frac{\binom{3}{2}\left[\binom{3}{1}\binom{3}{2}+\binom{9}{1}\binom{4}{2}\right]}{\binom{48}{3}}}_{\text{FH}} + \underbrace{5\frac{\left[\binom{12}{3}-16\right]}{\binom{48}{3}}}_{\text{F}} \\
&+ \underbrace{2\frac{\binom{3}{1}\left[\binom{9}{2}\binom{4}{1}^2+\binom{9}{1}\binom{4}{1}\binom{3}{1}^2+\binom{3}{2}\binom{3}{1}^2\right]+\binom{9}{1}\binom{4}{2}\left[\binom{8}{1}\binom{4}{1}+\binom{3}{1}^2\right]+\binom{3}{1}\binom{3}{2}\left[\binom{9}{1}\binom{4}{1}+\binom{2}{1}\binom{3}{1}\right]}{\binom{48}{3}}}_{\text{3-K}} \\
&+ \underbrace{\frac{2\left\{\binom{7}{2}\left[\binom{4}{1}^3-1\right]+\binom{7}{1}\binom{3}{1}\left[\binom{4}{1}\binom{3}{1}^2-1\right]\right\}+\binom{2}{2}\left\{\left[\binom{4}{1}^3-1\right]\binom{7}{1}+\left[\binom{4}{1}^2\binom{3}{1}-1\right]\binom{3}{1}\right\}}{\binom{48}{3}}}_{\text{HP}} \\
=\;& \frac{25,669}{\binom{48}{3}} \\
=\;& 1.48410 \qquad (5.7)
\end{aligned}
$$

$$
\begin{aligned}
E[R_T] \;=\;& \underbrace{200\frac{1}{\binom{48}{3}}}_{\text{5-K}} + \underbrace{100\frac{\binom{4}{1}}{\binom{48}{3}}}_{\text{RF}} + \underbrace{50\frac{\left[\binom{4}{1}^2-4\right]}{\binom{48}{3}}}_{\text{SF}} + \underbrace{3\frac{\binom{9}{1}\left[\binom{4}{1}^3-1\right]+\binom{7}{1}\left[\binom{4}{1}^2\binom{3}{1}-1\right]}{\binom{48}{3}}}_{\text{S}} \\
&+ \underbrace{20\frac{\binom{3}{2}\binom{45}{1}-\binom{9}{1}\binom{4}{3}+\binom{3}{1}}{\binom{48}{3}}}_{\text{4-K}} + \underbrace{7\frac{\binom{3}{2}\left[\binom{3}{1}\binom{3}{2}+\binom{9}{1}\binom{4}{2}\right]}{\binom{48}{3}}}_{\text{FH}} + \underbrace{5\frac{\left[\binom{12}{3}-16\right]}{\binom{48}{3}}}_{\text{F}} \\
&+ \underbrace{2\frac{\left\{\binom{3}{1}\left[\binom{9}{2}\binom{4}{1}^2+\binom{9}{1}\binom{4}{1}\binom{3}{1}^2+\binom{3}{2}\binom{3}{1}^2\right]+\binom{9}{1}\binom{4}{2}\left[\binom{8}{1}\binom{4}{1}+\binom{3}{1}^2\right]+\binom{3}{1}\binom{3}{2}\left[\binom{9}{1}\binom{4}{1}+\binom{2}{1}\binom{3}{1}\right]\right\}}{\binom{48}{3}}}_{\text{3-K}} \\
&+ \underbrace{\frac{\left\{\left[\binom{7}{2}-1\right]\left[\binom{4}{1}^3-1\right]+\binom{7}{1}\binom{3}{1}\left[\binom{4}{1}^2\binom{3}{1}-1\right]+\binom{3}{2}\left[\binom{4}{1}\binom{3}{2}^2-1\right]\right\}}{\binom{48}{3}}}_{\text{HP only with A}} \\
&+ \underbrace{\frac{\left\{\left[\binom{7}{2}-3\right]\left[\binom{4}{1}^3-1\right]+\binom{7}{1}\binom{3}{1}\left[\binom{4}{1}^2\binom{3}{1}-1\right]+\binom{3}{2}\left[\binom{4}{1}\binom{3}{1}^2-1\right]\right\}}{\binom{48}{3}}}_{\text{HP only with K}}
\end{aligned}
$$

$$+\underbrace{\frac{\binom{2}{2}\left\{\left[\binom{7}{1}-2\right]\left[\binom{4}{1}^3-1\right]+\left[\binom{4}{1}^2\binom{3}{1}-1\right]\binom{3}{1}\right\}}{\binom{48}{3}}}_{\text{HP with both A and K}}=\frac{25,491}{\binom{48}{3}}=1.47381 \qquad (5.8)$$

5.5 Overall Expected Return – Optimal Strategy

The common denominator expressed in (3.3) is slightly modified for Joker Wild because an extra card is added. In fact, the least common multiple is $\binom{53}{5}\binom{48}{5}5$ and the least common denominator is this number divided by 12. It follows that the overall expected return under optimal play is

$$\frac{24,727,653,272,220}{24,568,865,521,200} \approx 1.0064629663458800 \qquad (5.9)$$

Thus, the optimal Joker Wild players have a slight advantage (about 0.646 percent) over the house.

Table 5.4: The distribution of the payout R from a one-unit bet on Joker Wild. Assume maximum-coin bet and optimal drawing strategy

	R	number of ways	probability	probability $\times\binom{53}{5}\binom{48}{5}5/12$
royal flush (natural)	800	596,131,848	0.0000242637108126	49,677,654
five of a kind	200	2,293,355,592	0.0000933439759366	191,112,966
royal flush (joker)	100	2,556,304,788	0.0001040465130876	213,025,399
straight flush	50	14,124,168,708	0.0005748807854320	1,177,014,059
four of a kind	20	210,195,973,152	0.0085553796926694	17,516,331,096
full house	7	385,217,432,424	0.0156790891338309	32,101,452,702
flush	5	382,699,900,596	0.0155766207546610	31,891,658,383
straight	3	407,718,668,724	0.0165949326545903	33,976,555,727
three of a kind	2	3,290,682,627,144	0.1339371011780960	274,223,552,262
two pair	1	2,724,028,817,400	0.1108732031216290	227,002,401,450
pair of aces or kings	1	3,487,746,690,372	0.1419579869230220	290,645,557,531
others	0	13,661,005,450,452	0.5560291515562320	1,138,417,120,871
		24,568,865,521,200	1	2,047,405,460,100

Chapter 6

Conclusions

6.1 Jacks or Better

It is important to closely examine the contribution of each category of rank to the overall expected payout under optimal play. To do so, the level of magnitude in contribution of each rank is measured in ratio of optimal probability to pre-draw probability. This seems to be the most reasonable measurement to determine their contributions because each rank has different number of pre-draw hands. Therefore, we have provided a comparison table between pre-draw probability (equivalent to the probability in which there is no discard) and corresponding optimal probability. The overall expected return at pre-draw probability is 0.33687, equivalently inferring that the net gain is merely -0.66313 unit. But, when the optimal strategy is executed, the expected return significantly improves to 0.99544, or the net gain of -0.00456 which is close to breaking even. Then, the question of what accounts for the bulk of the improvement of 0.65857 needs to be answered. In fact, if players always make the correct play instead of settling for initial hands (pre-draw hands), probability of all the paying hands are improved. As the strategy is prioritized to achieve the higher ranks to maximize the return, it is relatively easy to see that the biggest improvements in ratio take place in more lucrative paying hands. To consider a few, the probability of a royal flush under optimal play is improved by more than factor of 16, four of a kind is improved to almost factor of 10, and full house by a factor of almost 8. As the hand-rank table guides players to choose a strategy that maximizes expected payout, we can expect that the probabilities of lower paying hands will now contribute to ones of higher paying hands as the their potential to be qualified for higher paying hands is realized by allowing better discards under optimal play. For example, if 2$\diamondsuit$-5$\diamondsuit$-9$\clubsuit$-9$\heartsuit$-Q$\spadesuit$ is dealt, then the optimal strategy for this particular hand is to hold 9$\clubsuit$-9$\heartsuit$ as 2-LP and discard 2$\diamondsuit$-5$\diamondsuit$-Q$\spadesuit$.

This allows the opportunity to parlay 2-LP to a higher hands such as 2P, 3K, FH, and 4K. By doing so, the expectation increases from 0 to 0.82368 ($E[R_{optimal}] = 0.82368 > E[R_{pre-draw}] = 0$). In a sense, 0.82368 may be described as "additional" probability that allows 2-LP to turn into these higher paying hands and increases the overall return. This additional figure is distributed according to its potential contributions to expected payout and then accumulated in the table. The concept of this "additional" probability added to the optimal probability is the reason why we see that all the hands improve under optimal strategy.

Table 6.1: Contribution to expected return under optimal play at Jacks or Better

result	pre-draw	optimal	$E[R_{pre-draw}]$	$E[R_{optimal}]$	ratio
royal flush	0.0000015391	0.0000247583	0.0012312617	0.0198066144	16.0864370747
straight flush	0.0000138517	0.0001093091	0.0006925847	0.0054654545	7.8913875976
four of a kind	0.0002400960	0.0023625457	0.0060024010	0.0590636421	9.8400027817
full house	0.0014405762	0.0115122073	0.0129651861	0.1036098660	7.9913905926
flush	0.0019654015	0.0110145110	0.0117924093	0.0660870658	5.6042038813
straight	0.0039246468	0.0112293672	0.0156985871	0.0449174690	2.8612427731
three of a kind	0.0211284514	0.0744486986	0.0633853541	0.2233460957	3.5236230631
two pair	0.0475390156	0.1292789025	0.0950780312	0.2585578050	2.7194274183
jacks or better	0.1300212393	0.2145850311	0.1300212393	0.2145850311	1.6503844475
others	0.7937251824	0.5454346692	0.0000000000	0.0000000000	0.6871832737

Let us help players understand the odds of each hand under optimal play in more practical setting. Two pair is responsible for approximately 25.97% (0.25856/0.99544) of the overall expected return and three of a kind and high pair are each accountable for slightly over 20% of overall expected return. Other notable hands are full house which is accountable for about 10% and royal flush for almost 2% of the overall mean expected return. Figure 6.2 shows the ratio of change in probability from pre-draw hand to the correct play under optimal strategy for each hand. A horizontal line is drawn to indicate a ratio of one, the point at which optimal play does not improve the expected return. Any paying hands that are above the dotted horizontal line, which is all the paying hands, are improved. Expressed as a ratio, the probability of royal flush improves by a factor of 16 and all the higher paying hands (full house and up) are improved by more than a factor of 5. This implies that initial hands, whose potential were not realized before any discards, now contribute to one or more of paying hands under the optimal strategy. Since all the hands are improved, we can see that the ratio of non-paying hands is less than 1 (0.68718).

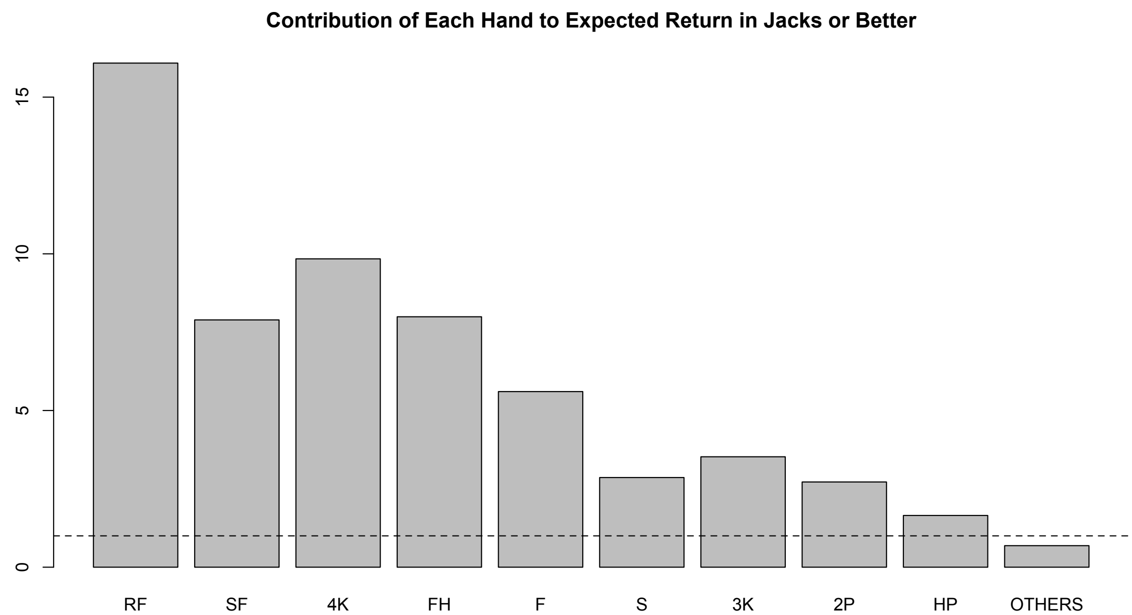

Figure 6.1: Contribution of each hand to overall expected return in Jacks or Better

6.2 Double Bonus

The pre-draw frenquencies of non-payings hands are equivalent to ones in Jacks or Better. While probability of non-paying hands is improved from 79.3725% to 54.54346% in Jacks or Better, it is only improved from 79.3725% to 56.71361% in Double Bonus although it is still significant. This means there are more hands in Jacks or Better that result in payable hands under optimal play than Double Bonus. Interestingly, Double Bonus has 0.63132% (1.001752 – 0.995439) higher expected return than Jacks or Better. Known as an "ugly" game as Dancer (2005) mentioned in Section 4.1, Table 6.2 itself clearly explains why it is essential for players to hit quads to benefit from the optimal strategy. The magnitude of improvement in 4K under optimal strategy is significant to be mentioned (1.5698% to 15.4114% in quads in terms of expectation) in Double Bonus. Other than quads, each rank seems to exhibit a similar degree of improvement.

Table 6.2: Contribution to expected return under optimal play at Double Bonus

result	pre-draw	optimal	$E[R_{pre-draw}]$	$E[R_{optimal}]$	ratio
royal flush	0.0000015391	0.0000208125	0.0012312617	0.0166500045	13.5227173962
straight flush	0.0000138517	0.0001131046	0.0006925847	0.0056552282	8.1653955383
four aces	0.0000184689	0.0001987906	0.0029550282	0.0318064883	10.7635144227
four 2's, 3's, or 4's	0.0000554068	0.0005240628	0.0044325422	0.0419250243	9.4584601800
other four of a kind	0.0001662203	0.0016076668	0.0083110167	0.0803833381	9.6719018741
full house	0.0014405762	0.0111898901	0.0144057623	0.1118989009	7.7676487035
flush	0.0019654015	0.0149533473	0.0137578108	0.1046734313	7.6082912268
straight	0.0039246468	0.0150194092	0.0196232339	0.0750970459	3.8269454593
three of a kind	0.0211284514	0.0721994483	0.0633853541	0.2165983449	3.4171670705
two pair	0.0475390156	0.1246583705	0.0475390156	0.1246583705	2.6222328941
jacks or better	0.1300212393	0.1923790465	0.1300212393	0.1923790465	1.4795970845
others	0.7937251824	0.5671360509	0.0000000000	0.0000000000	0.7145244519

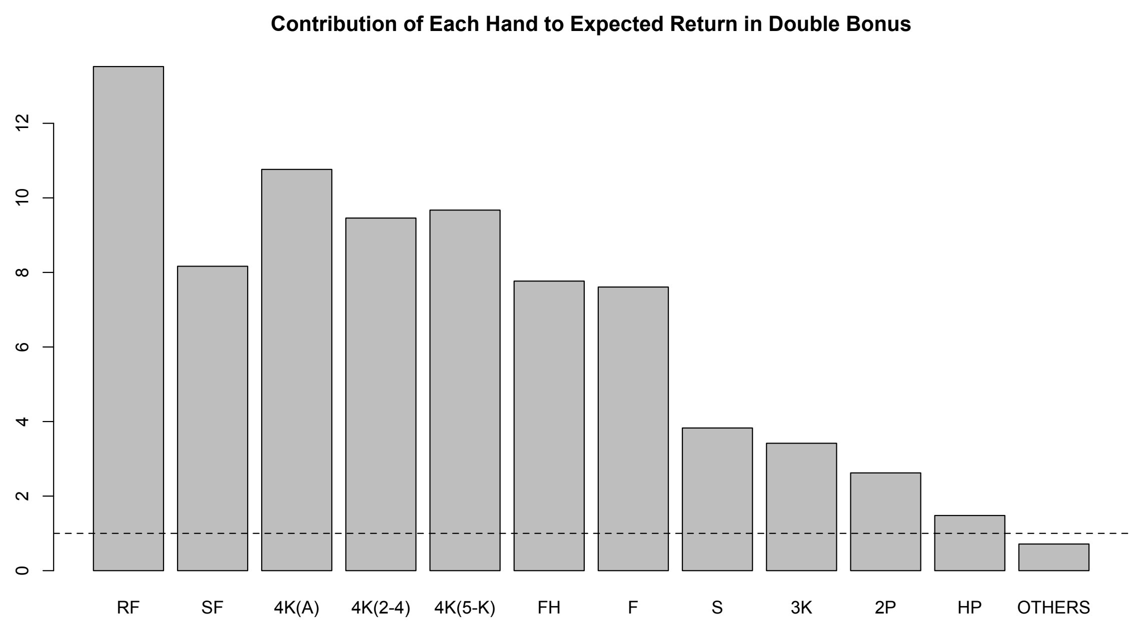

Figure 6.2: Contribution of each hand to overall expected return in Double Bonus

6.3 Joker Wild

Table 6.3: Contribution to expected return under optimal play at Joker Wild

result	pre-draw	optimal	$E[R_{pre-draw}]$	$E[R_{optimal}]$	ratio
royal flush (natural)	0.0000013939	0.0000242637	0.0011151050	0.0194109687	17.4073017408
five of a kind	0.0000045301	0.0000933440	0.0009060228	0.0186687952	20.6052159681
royal flush (joker)	0.0000069694	0.0001040465	0.0006969406	0.0104046513	14.9290358955
straight flush	0.0000627247	0.0005748808	0.0031362327	0.0287440393	9.1651487041
four of a kind	0.0010872273	0.0085553797	0.0217445469	0.1711075939	7.8689887094
full house	0.0022831774	0.0156790891	0.0159822420	0.1097536239	6.8672232755
flush	0.0027194622	0.0155766208	0.0135973112	0.0778831038	5.7278312315
straight	0.0071547923	0.0165949327	0.0214643768	0.0497847980	2.3194150261
three of a kind	0.0478380031	0.1339371012	0.0956760063	0.2678742024	2.7998054356
two pair	0.0430542028	0.1108732031	0.0430542028	0.1108732031	2.5752004654
aces or kings	0.0916323569	0.1419579869	0.0916323569	0.1419579869	1.5492124375
others	0.8041551599	0.5560291516	0.0000000000	0.0000000000	0.6914451082

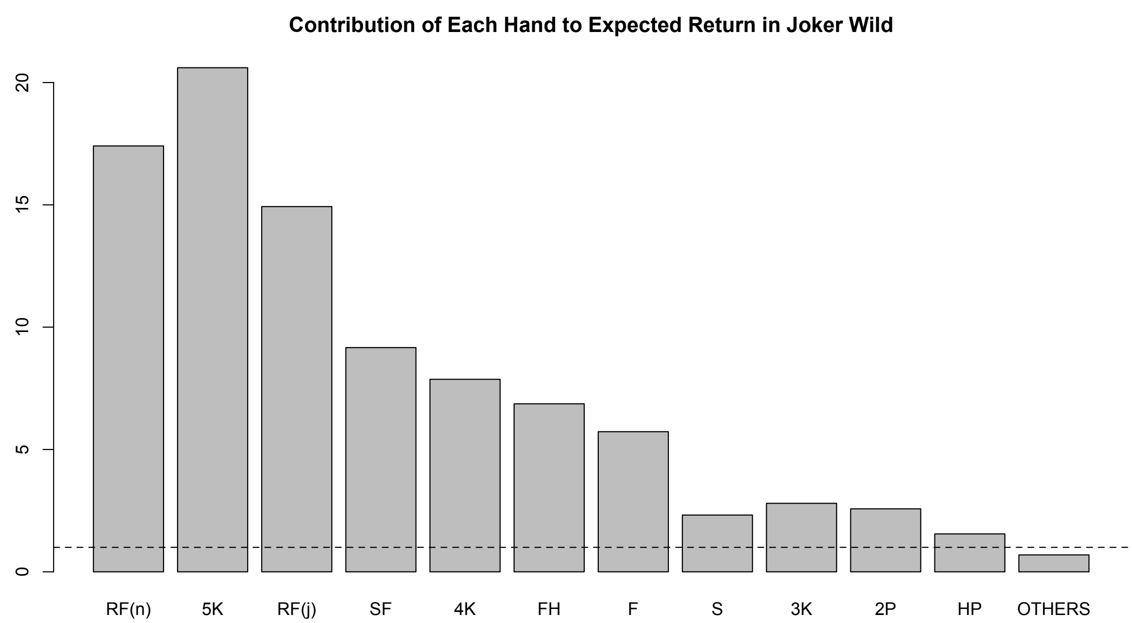

Figure 6.3: Contribution of each hand to overall expected return in Joker Wild

It is expected that players will encounter nearly 50% of hands resulting in three of a kind, two pair, or pair of kings or better under optimal play. Most of the return comes from low ranks. For the fact that these following hands pay at most 2 for 1. players might become greedy enough not to attempt to achieve lower paying hands. However, players should not force a hand to allow a chance for something better when the optimal strategy is to settle for low ranks. There are many expected occurrences of these hands under optimal strategy that players should rely on the long run behaviour of the hands.

We close with the remark that the natural royal flush and five of a kind are improved by more than a factor of 17. These are the biggest improvements that we have seen in all three video poker games.

Appendix A

Jacks or Better

Rank	Description	E[R]
1**	5-RF: T,J,Q,K,A (1,1,1,1,1)	800.00000
2**	5-SF: 2,3,4,5,6 (1,1,1,1,1)	50.00000
3**	5-4K: 2,2,2,2,3 (1,2,3,4,1)	25.00000
4	4-RF: *,J,Q,K,A (1,1,1,1,1)	18.42553 (2-9)
	4-RF: *,J,Q,K,A (1,2,2,2,2)	18.55319 (2-9)
		18.46809 (T), 18.53191 (J-A)
	4-RF: *,T,Q,K,A (1,1,1,1,1)	18.36170 (2-9)
	4-RF: *,T,Q,K,A (1,2,2,2,2)	18.48936 (2-10)
		18.40426 (J), 18.46809 (Q-A)
	4-RF: *,T,J,K,A (1,1,1,1,1)	18.36170 (2-9)
	4-RF: *,T,J,K,A (1,2,2,2,2)	18.48936 (2-T)
		18.40426 (Q), 18.46809 (J,K,A)
	4-RF: *,T,J,Q,A (1,1,1,1,1)	18.36170 (2-9)
	4-RF: *,T,J,Q,A (1,2,2,2,2)	18.48936 (2-T)
		18.40426 (K), 18.46809 (J,Q,A)
	4-RF: *,T,J,Q,K (1,1,1,1,1)	19.55319 (2-8)
	4-RF: *,T,J,Q,K (1,2,2,2,2)	19.68085 (2-8,T)
		19.59574 (9,A), 19.65957 (J,Q,K)
5**	5-FH: 2,2,2,3,3 (1,2,3,1,2)	9.00000
6**	5-F: 2,4,5,7,T (1,1,1,1,1)	6.00000
7**	3-3K: 2,2,2,3,4 (1,2,3,1,2)	4.30250
8**	5-S: 2,3,4,5,6 (1,2,3,4,1)	4.00000
9	4-SF (outside): 2,3,4,5,T (1,1,1,1,2), 0H	3.53191
	4-SF (outside): 8,9,T,J,K (1,1,1,1,2), 1H	3.59574
	4-SF (outside): 8,9,T,J,J (1,1,1,1,2), 1H, 2-HP	3.57447
	4-SF (outside): 9,T,J,Q,A (1,1,1,1,2), 2H	3.65957
	4-SF (outside): 9,T,J,Q,Q (1,1,1,1,2), 2H, 2-HP	3.63830
10**	4-2P: 2,2,3,3,4 (1,2,1,2,1)	2.59574

Rank	**Description**	**E[R]**
11	4-SF (inside): 2,3,4,6,8 (1,1,1,1,2), 0H	2.34043
	4-SF (inside): 7,8,10,J,A (1,1,1,1,2), 1H	2.40426
	4-SF (inside): 7,8,10,J,J (1,1,1,1,2), 1H, 2-HP	2.38298
	4-SF (inside): 8,9,J,Q,A (1,1,1,1,2), 2H	2.46809
	4-SF (inside): 8,9,J,Q,Q (1,1,1,1,2), 2H, 2-HP	2.44681
	4-SF (inside): 7,9,J,Q,K (1,2,2,2,2), 3H	2.53191
	4-SF (inside): 9,J,Q,K,K (1,1,1,1,2), 3H, 2-HP	2.51064
12**	2-HP: Q,Q,5,7,T (1,2,1,2,1)	1.53654
13**	$AHTx$+K,Q,J or 10	
	,T,,K,A (1.1,2,1,1)	1.27660
	,T,Q,,A (1.1,1,2,1)	1.27660
	,T,J,,A (1.1,1,2,1)	1.27660
14	KQJ	
	3-RF: KQJ: 3,8,J,Q,K (1,1,2,2,2)	1.53284
	3-RF: KQJ: 3,J,Q,K,A (1,2,2,2.3), shp (inside)	1.51526
	3-RF: KQJ: 3,9,J,Q,K (1,1,2,2,2), sp (inside)	1.51804
	3-RF: KQJ: 3,T,J,Q,K (1,1,2,2,2), sp (outside)	1.50324
	3-RF: KQJ: 3,8,J,Q,K (1,2,2,2,2), fp	1.48289
	3-RF: KQJ: 8,8,J,Q,K (1,2,3,3,3), 2-LP (2-8)	1.53284
	3-RF: KQJ: 9,J,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.50046
	3-RF: KQJ: 3,J,Q,K,A (1,1,1,1,2), shp (inside), fp	1.46531
‘	3-RF: KQJ: 3,9,J,Q,K (1,2,1,1,1), sp (inside), fp	1.46809
	3-RF: KQJ: 3,T,J,Q,K (1,2,1,1,1), sp (outside), fp	1.45328
	3-RF: KQJ: 9,9,J,Q,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.50324
	3-RF: KQJ: T,T,J,Q,K (1,2,3,3,3), sp (outside), 2-LP (T)	1.47364
	3-RF: KQJ: 8,8,J,Q,K (1,2,2,2,2), fp, 2-LP (2-8)	1.48289
	QJT	
	3-RF: QJT: 3,7,T,J,Q (1,1,2,2,2)	1.52914
	3-RF: QJT: 3,T,J,Q,A (1,2,2,2,1), shp (inside)	1.51156
	3-RF: QJT: 3,T,J,Q,K (1,2,2,2,1), shp (outside)	1.49676
	3-RF: QJT: 3,8,T,J,Q (1,1,2,2,2), sp (inside)	1.51434
	3-RF: QJT: 3,9,T,J,Q (1,1,2,2,2), sp (outside)	1.49954
	3-RF: QJT: 3,7,T,J,Q (1,2,2,2,2), fp	1.47919
	3-RF: QJT: 7,7,T,J,Q (1,2,3,3,3), 2-LP (2-7)	1.52914
	3-RF: QJT: 7,T,T,J,Q (1,2,3,3,3), 2-LP (T)	1.51804
	3-RF: QJT: 8,T,J,Q,A (1,2,2,2,3), shp (inside), sp (inside)	1.49676
	3-RF: QJT: 8,T,J,Q,K (1,2,2,2,3), shp (outside), sp (inside)	1.48196
	3-RF: QJT: 9,T,J,Q,A (1,2,2,2,3), shp (inside), sp (outside)	1.48196
	3-RF: QJT: 3,T,J,Q,A (1,1,1,1,2), shp (inside), fp	1.46161
	3-RF: QJT: 3,T,J,Q,K (1,1,1,1,2), shp (outside), fp	1.44681
	3-RF: QJT: T,T,J,Q,A (1,2,2,2,3), shp (inside), 2-LP (T)	1.50046
	3-RF: QJT: T,T,J,Q,K (1,2,2,2,3), shp (outside), 2-LP (T)	1.48566
	3-RF: QJT: 3,8,T,J,Q (1,2,1,1,1), sp (inside), fp	1.46438
	3-RF: QJT: 3,9,T,J,Q (1,2,1,1,1), sp (outside), fp	1.44958
	3-RF: QJT: 8,8,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (8)	1.49954
	3-RF: QJT: 9,9,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (9)	1.46994
	3-RF: QJT: 8,T,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (T)	1.50324
	3-RF: QJT: 9,T,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (T)	1.48844
	3-RF: QJT: 7,7,T,J,Q (1,2,1,1,1), fp, 2-LP (2-7)	1.47919
	3-RF: QJT: 7,T,T,J,Q (1,1,2,1,1), fp, 2-LP (T)	1.46809

Rank	Description	E[R]
14	*KJT*	
	3-RF: KJT: 3,8,T,J,K (1,1,2,2,2)	1.43293
	3-RF: KJT: 3,T,J,K,A (1,2,2,2,3), shp (inside)	1.41536
	3-RF: KJT: 8,T,J,Q,K (1,2,2,3,2), shp (outside)	1.40056
	3-RF: KJT: 3,9,T,J,K (1,1,2,2,2), sp (inside)	1.41813
	3-RF: KJT: 3,8,T,J,K (1,2,2,2,2), fp	1.38298
	3-RF: KJT: 8,8,T,J,K (1,2,3,3,3), 2-LP (2-8)	1.43293
	3-RF: KJT: 8,T,T,J,K (1,2,3,3,3), 2-LP (T)	1.42183
	3-RF: KJT: 9,T,J,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.40056
	3-RF: KJT: 3,T,J,K,A (1,1,1,1,2), shp (inside), fp	1.36540
	3-RF: KJT: 8,T,J,Q,K (1,1,1,2,1), shp (outside), fp	1.35060
	3-RF: KJT: 3,9,T,J,K (1,2,1,1,1), sp (inside), fp	1.36818
	3-RF: KJT: T,T,J,K,A (1,2,3,3,3), shp (inside), 2-LP (T)	1.40426
	3-RF: KJT: T,T,J,Q,K (1,2,2,3,2), shp (outside), 2-LP (T)	1.38945
	3-RF: KJT: 9,9,T,J,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.40333
	3-RF: KJT: 9,T,T,J,K (1,2,3,3,3), sp (inside), 2-LP (T)	1.40703
	3-RF: KJT: 8,8,T,J,K (1,2,1,1,1), fp, 2-LP (2-8)	1.38298
	3-RF: KJT: 8,T,T,J,K (1,1,2,1,1), fp, 2-LP (T)	1.37188
	KQT	
	3-RF: KQT: 3,8,T,Q,K (1,1,2,2,2)	1.43293
	3-RF: KQT: 3,T,Q,K,A (1,2,2,2,3), shp (inside)	1.41536
	3-RF: KQT: 3,T,J,Q,K (1,2,3,2,2), shp (outisde)	1.40056
	3-RF: KQT: 3,9,T,Q,K (1,1,2,2,2), sp (inside)	1.41813
	3-RF: KQT: 3,8,T,Q,K (1,2,2,2,2), fp	1.38298
	3-RF: KQT: 8,8,T,Q,K (1,2,3,3,3), 2-LP (2-8)	1.43293
	3-RF: KQT: 8,T,T,Q,K (1,2,3,3,3), 2-LP (T)	1.42183
	3-RF: KQT: 9,T,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.40056
	3-RF: KQT: 3,T,Q,K,A (1,1,1,1,2), shp (inside), fp	1.36540
	3-RF: KQT: 3,T,J,Q,K (1,1,2,1,1), shp (outisde), fp	1.35060
	3-RF: KQT: T,T,Q,K,A (1,2,2,2,3), shp (inside), 2-LP (T)	1.40426
	3-RF: KQT: T,T,J,Q,K (1,2,2,2,3), shp (outisde), 2-LP (T)	1.38945
	3-RF: KQT: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.40333
	3-RF: KQT: 9,T,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (T)	1.40703
	3-RF: KQT: 3,9,T,Q,K (1,2,1,1,1), sp (inside), fp	1.36818
	3-RF: KQT: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP	1.40333
	3-RF: KQT: 8,8,T,Q,K (1,2,1,1,1), fp, 2-LP (2-8)	1.38298
	3-RF: KQT: 8,T,T,Q,K (1,1,2,1,1), fp, 2-LP (T)	1.37188
	AQJ	
	3-RF: AQJ: 3,8,J,Q,A (1,1,2,2,2)	1.43663
	3-RF: AQJ: 8,J,Q,K,A (1,2,2,3,2), shp (inside)	1.41906
	3-RF: AQJ: 3,T,J,Q,A (1,2,3,3,3), sp (inside)	1.42183
	3-RF: AQJ: 3,8,J,Q,A (1,2,2,2,2), fp	1.38668
	3-RF: AQJ: 9,9,J,Q,A (1,2,3,3,3), 2-LP (2-9)	1.43663
	3-RF: AQJ: 3,J,Q,K,A (1,1,1,2,1), shp (inside), fp	1.36910
	3-RF: AQJ: 3,T,J,Q,A (1,2,1,1,1), sp (inside), fp	1.37188
	3-RF: AQJ: T,T,J,Q,A (1,2,3,3,3), sp (inside), 2-LP	1.40703
	3-RF: AQJ: 9,9,J,Q,A (1,2,1,1,1), fp, 2-LP (2-9)	1.38668

Rank	Description	E[R]
14	*AKJ*	
	3-RF: *AKJ*: 8,9,J,K,A (1,1,2,2,2)	1.43663
	3-RF: *AKJ*: 8,J,Q,K,A (1,2,3,3,3), shp (inside)	1.41906
	3-RF: *AKJ*: 3,T,J,K,A (1,2,3,3,3), sp (inside)	1.42183
	3-RF: *AKJ*: 8,9,J,K,A (1,2,2,2,2), fp	1.38668
	3-RF: *AKJ*: 9,9,J,K,A (1,1,2,2,2), 2-LP (2-9)	1.43663
	3-RF: *AKJ*: 8,J,Q,K,A (1,1,3,1,1), shp (inside), fp	1.36910
	3-RF: *AKJ*: 3,T,J,K,A (1,2,1,1,1), sp (inside), fp	1.37188
	3-RF: *AKJ*: T,T,J,K,A (1,2,3,3,3), sp (inside), 2-LP	1.40703
	3-RF: *AKJ*: 9,9,J,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.38668
	AKQ	
	3-RF: *AKQ*: 8,9,Q,K,A (1,1,2,2,2)	1.43663
	3-RF: *AKQ*: 8,J,Q,K,A (1,1,2,2,2), shp (inside)	1.41906
	3-RF: *AKQ*: 8,T,Q,K,A (1,1,2,2,2), sp (inside)	1.42183
	3-RF: *AKQ*: 8,9,Q,K,A (1,2,2,2,2), fp	1.38668
	3-RF: *AKQ*: 9,9,Q,K,A (1,2,2,2,2), 2-LP (2-9)	1.43663
	3-RF: *AKQ*: 8,J,Q,K,A (1,2,1,1,1), shp (inside), fp	1.36910
	3-RF: *AKQ*: 8,T,Q,K,A (1,2,1,1,1), sp (inside), fp	1.37188
	3-RF: *AKQ*: T,T,Q,K,A (1,2,3,3,3), sp (inside), 2-LP (T)	1.40703
	3-RF: *AKQ*: 9,9,Q,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.38668
	AKT	
	3-RF: *AKT*: 8,9,T,K,A (1,1,2,2,2)	1.33673
	3-RF: *AKT*: 8,T,Q,K,A (1,2,3,2,2), shp (inside)	1.31915
	3-RF: *AKT*: 8,T,J,K,A (1,2,3,2,2), shp (inside)	1.31915
	3-RF: *AKT*: 8,9,T,K,A (1,2,2,2,2), fp	1.28677
	3-RF: *AKT*: 9,9,T,K,A (1,2,3,3,3), 2-LP (2-9)	1.33673
	3-RF: *AKT*: 9,T,T,K,A (1,2,3,3,3), 2-LP (T)	1.32562
	3-RF: *AKT*: T,T,Q,K,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.30805
	3-RF: *AKT*: T,T,J,K,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.30805
	3-RF: *AKT*: 9,9,T,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.28677
	AQT	
	3-RF: *AQT*: 3,9,T,Q,A (1,1,2,2,2)	1.33673
	3-RF: *AQT*: 8,T,Q,K,A (1,2,2,3,2), shp (inside)	1.31915
	3-RF: *AQT*: 8,T,J,Q,A (1,2,3,2,2), shp (inside)	1.31915
	3-RF: *AQT*: 3,9,T,Q,A (1,2,2,2,2), fp	1.28677
	3-RF: *AQT*: 9,9,T,Q,A (1,2,3,3,3), 2-LP (2-9)	1.33673
	3-RF: *AQT*: 9,T,T,Q,A (1,2,3,3,3), 2-LP (T)	1.32562
	3-RF: *AQT*: T,T,Q,K,A (1,2,1,3,1), shp (inside), 2-LP (T)	1.30805
	3-RF: *AQT*: T,T,J,Q,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.30805
	3-RF: *AQT*: 9,9,T,Q,A (1,2,1,1,1), fp, 2-LP (2-9)	1.28677
	AJT	
	3-RF: *AJT*: 3,9,T,J,A (1,1,2,2,2)	1.33673
	3-RF: *AJT*: 3,T,J,K,A (1,2,2,3,2), shp (inside)	1.31915
	3-RF: *AJT*: 3,T,J,Q,A (1,2,2,3,2), shp (inside)	1.31915
	3-RF: *AJT*: 3,9,T,J,A (1,2,2,2,2), fp	1.28677
	3-RF: *AJT*: 9,9,T,J,A (1,2,3,3,3), 2-LP (2-9)	1.33673
	3-RF: *AJT*: 3,T,T,J,A (1,1,2,2,2), 2-LP (T)	1.32562
	3-RF: *AJT*: T,T,J,K,A (1,2,2,3,2), shp (inside), 2-LP (T)	1.30805
	3-RF: *AJT*: T,T,J,Q,A (1,2,2,3,2), shp (inside), 2-LP (T)	1.30805
	3-RF: *AJT*: 9,9,T,J,A (1,2,1,1,1), fp, 2-LP (2-9)	1.28677

Rank	Description	E[R]
15	4-F: 4,7,9,J,Q (1,2,2,2,2), 2H	1.27660
	4-F: 4,7,9,T,K (1,2,2,2,2), 1H	1.21277
	4-F: 3,4,7,8,9 (1,1,1,2,1), 0H	1.14894
16**	4-S: KQJT: 3,T,J,Q,K (1,2,3,4,1)	0.87234
17**	2-LP: 6,6,8,9,T (1,2,3,3,3)	0.82368
18	4-S: 5432–QJT9: 2,9,T,J,Q (1,2,3,4,1), 2H QJT9	0.80851
	4-S: 5432–QJT9: 2,8,9,T,J (1,2,3,4,1), 1H JT98	0.74468
	4-S: 5432–QJT9: 2,3,4,5,T (1,2,3,4,1), 0H 5432-T987	0.68085
19	2H, 2S (inside)	
	3-SF: $s + h \geq 3$: 4,5,9,J,Q (1,2,3,3,3)	0.73913
	3-SF: $s + h \geq 3$: 4,9,J,Q,K (1,2,2,2,3), shp (inside)	0.72155
	3-SF: $s + h \geq 3$: 4,9,J,Q,A (1,2,2,2,3), hp	0.73636
	3-SF: $s + h \geq 3$: 4,8,9,J,Q (1,2,3,3,3), sp (inside)	0.72433
	3-SF: $s + h \geq 3$: 9,J,Q,K,A (1,1,1,2,3), shp (inside), hp	0.71878
	3-SF: $s + h \geq 3$: 8,9,J,Q,K (1,2,2,2,3), shp (inside), sp (inside)	0.70675
	3-SF: $s + h \geq 3$: 8,9,J,Q,A (1,2,2,2,3), hp, sp (inside)	0.72155
	1H, 3S (outside)	
	3-SF: $s + h \geq 3$: 4,5,9,10,J (1,2,3,3,3)	0.73543
	3-SF: $s + h \geq 3$: 4,9,10,J,K (1,2,2,2,3), shp (inside)	0.71785
	3-SF: $s + h \geq 3$: 5,9,10,J,A (1,2,2,2,3), hp	0.73265
	3-SF: $s + h \geq 3$: 4,7,9,10,J (1,2,3,3,3), sp (inside)	0.72063
	3-SF: $s + h \geq 3$: 9,10,J,K,A (1,1,1,2,3), shp (inside), hp	0.71508
	3-SF: $s + h \geq 3$: 7,9,10,J,K (1,2,2,2,3), shp (inside), sp (inside)	0.70305
	3-SF: $s + h \geq 3$: 7,9,10,J,A (1,2,2,2,3), hp, sp (inside)	0.71785
	2H, 1S (double-inside)	
	3-SF: $s + h \geq 3$: 4,5,9,J,K (1,2,3,3,3)	0.64292
	3-SF: $s + h \geq 3$: 4,9,J,Q,K (1,2,2,3,2), shp (inside)	0.62535
	3-SF: $s + h \geq 3$: 4,9,J,K,A (1,2,2,2,3), hp	0.64015
	3-SF: $s + h \geq 3$: 4,9,T,J,K (1,2,3,2,2), sp (inside)	0.62812
	3-SF: $s + h \geq 3$: 9,J,Q,K,A (1,1,2,1,3), shp (inside), hp	0.62257
	3-SF: $s + h \geq 3$: 9,T,J,K,A (1,2,1,1,3), hp, sp (inside)	0.62535
	1H, 2S (inside)	
	3-SF: $s + h \geq 3$: 4,5,9,10,Q (1,2,3,3,3)	0.63922
	3-SF: $s + h \geq 3$: 4,9,10,Q,K (1,2,2,2,3), shp (inside)	0.62165
	3-SF: $s + h \geq 3$: 4,9,10,Q,A (1,2,2,2,3), hp	0.63645
	3-SF: $s + h \geq 3$: 4,8,9,10,Q (1,2,3,3,3), sp (inside)	0.62442
	3-SF: $s + h \geq 3$: 9,10,Q,K,A (1,1,1,2,3), shp (inside), hp	0.61887
	3-SF: $s + h \geq 3$: 8,9,10,Q,K (1,2,2,2,3), shp (inside), sp (inside)	0.60685
	3-SF: $s + h \geq 3$: 8,9,10,Q,A (1,2,2,2,3), hp, sp (inside)	0.62165
	0H, 3S (outside)	
	3-SF: $s + h \geq 3$: 4,5,8,9,10 (1,2,3,3,3)	0.63552
	3-SF: $s + h \geq 3$: 4,8,9,10,Q (1,2,2,2,3), shp (inside)	0.61795
	3-SF: $s + h \geq 3$: 4,8,9,10,K (1,2,2,2,3), hp	0.63275
	3-SF: $s + h \geq 3$: 4,6,8,9,10 (1,2,3,3,3), sp (inside)	0.62072
	3-SF: $s + h \geq 3$: 8,9,10,Q,A (1,1,1,2,3), shp (inside) hp	0.61517
	3-SF: $s + h \geq 3$: 6,8,9,10,Q (1,2,2,2,3), shp (inside), sp (inside)	0.60315
	3-SF: $s + h \geq 3$: 6,8,9,10,K (1,2,2,2,3), hp, sp (outside)	0.61795
20**	4-S: AKQJ if QJ fp or 9p: 7,J,Q,K,A (1,1,1,2,3)	0.59574
	4-S: AKQJ if QJ fp or 9p: 9,J,Q,K,A (1,2,2,3,4)	0.59574

Rank	Description	E[R]
21	2-RF: QJ: 2,5,7,J,Q (1,2,3,4,4)	0.62455
	2-RF: QJ: 6,7,J,Q,A (1,2,3,3,4), shp (double-inside)	0.61394
	2-RF: QJ: 6,7,J,Q,K (1,2,3,3,4), shp (inside)	0.60999
	2-RF: QJ: 7,J,Q,K,A (1,2,2,3,4), 2shp (double-inside+inside)	0.60037
	2-RF: QJ: 6,7,8,J,Q,(1,2,3,4,4), sp (double-inside)	0.62060
	2-RF: QJ: 6,7,9,J,Q,(1,2,3,4,4), sp (inside)	0.61665
	2-RF: QJ: 6,7,T,J,Q,(1,2,3,4,4), sp (outside)	0.61270
	2-RF: QJ: 6,8,T,J,Q,(1,2,3,4,4), 2sp (double-inside+outside)	0.60974
	2-RF: QJ: 6,8,9,J,Q,(1,2,3,4,4), 2sp (inside+double-inside)	0.61369
	2-RF: QJ: 2,5,7,J,Q (1,2,3,3,3), fp	0.60789
	2-RF: QJ: 6,8,J,Q,A (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.60999
	2-RF: QJ: 6,9,J,Q,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.60604
	2-RF: QJ: 6,10,J,Q,A (1,2,3,3,4), shp (double-inside), sp (outside)	0.60308
	2-RF: QJ: 6,8,J,Q,K (1,2,3,3,4), shp (inside), sp (double-inside)	0.60604
	2-RF: QJ: 6,9,J,Q,K (1,2,3,3,4), shp (inside), sp (inside)	0.60308
	2-RF: QJ: 8,9,J,Q,A (1,2,3,3,4), shp (double-inside), 2sp (double-inside+inside)	0.60308
	2-RF: QJ: 8,10,J,Q,A (1,2,3,3,4), shp (double-inside), 2sp (outside+double-inside)	0.60012
	2-RF: QJ: 8,9,J,Q,K (1,2,3,3,4), shp (inside), 2sp (double-inside+inside)	0.60012
	2-RF: QJ: 6,7,J,Q,A (1,3,3,3,4), shp (double-inside), fp	0.59729
	2-RF: QJ: 6,7,J,Q,K (1,3,3,3,4), shp (inside), fp	0.59334
	2-RF: QJ: 8,J,Q,K,A (1,2,2,3,4), 2shp (double-inside+inside), sp (double-inside)	0.59642
	2-RF: QJ: 6,8,J,Q,A (1,2,1,1,3), shp (double-inside), sp (double-inside), fp	0.59334
	2-RF: QJ: 6,9,J,Q,A (1,2,1,1,3), shp (double-inside), sp (inside), fp	0.58939
	2-RF: QJ: 6,10,J,Q,A (1,2,1,1,3), shp (double-inside), sp (outside), fp	0.58643
	2-RF: QJ: 6,8,J,Q,K (1,2,1,1,3), shp (inside), sp (double-inside), fp	0.58939
	2-RF: QJ: 6,9,J,Q,K (1,2,1,1,3), shp (inside), sp (inside), fp	0.58643
22**	4-S: AKQJ: 4,J,Q,K,A (1,2,3,4,1)	0.59574
23	2-RF: KH: 5,7,8,Q,K (1,2,3,4,4)	0.60629
	2-RF: KH: 5,7,Q,K,A (1,2,3,3,4), shp (double-inside)	0.59568
	2-RF: KH: 5,7,J,Q,K (1,2,3,4,4), shp (inside)	0.59174
	2-RF: KH: 5,8,T,Q,K (1,2,3,4,4), sp (inside)	0.59840
	2-RF: KH: 5,8,9,Q,K (1,2,3,4,4), sp (double-inside)	0.60234
	2-RF: KH: 5,9,T,Q,K (1,2,3,4,4), 2sp (double-inside+inside)	0.59544
	2-RF: KH: 5,7,8,Q,K (1,2,3,3,3), fp	0.58964
	2-RF: KH: 5,9,Q,K,A (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.59174
	2-RF: KH: 5,T,Q,K,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.58878
	2-RF: KH: 5,9,J,Q,K (1,2,3,4,4), shp (inside), sp (double-inside)	0.58878
	2-RF: KH: 5,7,Q,K,A (1,2,1,1,3), shp (double-inside), fp	0.57903
	2-RF: KH: 5,7,J,Q,K (1,2,3,2,2), shp (inside), fp	0.57508
	2-RF: KH: 5,8,T,Q,K (1,2,3,1,1), sp (inside), fp	0.59175
	2-RF: KH: 5,8,9,Q,K (1,2,3,1,1), sp (double-inside), fp	0.58569
	2-RF: KH: 5,9,T,Q,K (1,2,3,1,1), 2sp (double-inside+inside), fp	0.57879
	2-RF: KH: 5,9,Q,K,A (1,2,1,1,3), shp (double-inside), sp (double-inside), fp	0.57508
	2-RF: KH: 5,T,Q,K,A (1,2,1,1,3), shp (double-inside), sp (inside), fp	0.57212
	2-RF: KH: 5,9,J,Q,K (1,2,3,1,1), shp (inside), sp (double-inside), fp	0.57212
24	2-RF: AH: 5,8,9,K,A (1,2,3,4,4)	0.58804
	2-RF: AH: 5,8,Q,K,A (1,2,3,4,4), shp (double-inside)	0.57743
	2-RF: AH: 5,8,T,K,A (1,2,3,4,4), sp (double-inside)	0.58409
	2-RF: AH: 5,T,Q,K,A (1,2,3,4,4), shp (double-inside), sp (double-inside)	0.57447

Rank	Description	E[R]
25	1H, 1S	
	3-SF: $s + h = 2$, no sp: 2,8,9,T,K (1,1,2,2,2)	0.54302
	3-SF: $s + h = 2$, no sp: 7,8,J,K,A (1,1,1,2,3), 2hp	0.53747
	3-SF: $s + h = 2$, no sp: 2,9,T,K,A (1,2,2,2,3), hp	0.54024
	0H, 2S	
	3-SF: $s + h = 2$, no sp: 2,3,6,8,9 (1,1,2,2,2)	0.53932
	3-SF: $s + h = 2$, no sp: 2,6,8,9,K (1,2,2,2,3), hp	0.53654
	3-SF: $s + h = 2$, no sp: 6,8,9,K,A (1,1,1,2,2), 2hp	0.53377
26**	4-S: AHHT or KQJ9: 2,T,Q,K,A (1,1,2,3,4)	0.53191
	4-S: AHHT or KQJ9: 7,9,J,Q,K (1,1,2,3,4)	0.53191
27	1H, 1S	
	3-SF: $s + h = 2$, sp: 2,9,T,Q,K (1,2,2,3,2), shp (inside)	0.52544
	3-SF: $s + h = 2$, sp: 2,9,T,J,K (1,2,2,3,2), shp (inside)	0.52544
	3-SF: $s + h = 2$, sp: 2,8,9,T,Q (1,2,2,3,2) sp (inside)	0.52821
	3-SF: $s + h = 2$, sp: 8,9,J,Q,A (1,1,2,1,3), shp (inside), hp	0.52266
	0H, 2S	
	3-SF: $s + h = 2$, sp: 2,7,8,T,J (1,2,2,2,3), shp (inside)	0.52174
	3-SF: $s + h = 2$, sp: 7,8,T,J,A (1,1,1,2,3), shp (inside), hp	0.51896
	3-SF: $s + h = 2$, sp: 6,7,8,T,J (1,2,2,2,3), shp (inside), sp (inside)	0.50694
	3-SF: $s + h = 2$, sp: 2,6,8,9,T (1,2,2,2,3), sp (inside)	0.52451
	3-SF: $s + h = 2$, sp: 5,6,8,9,A (1,2,2,2,3), hp, sp (inside)	0.52174
	3-SF: $s + h = 2$, sp: 6,8,9,K,A (1,1,1,2,3), 2hp	0.53377
28**	3-S: KQJ: 2,5,J,Q,K (1,1,2,3,4)	0.51526
29	2-S: QJ: 2,5,6,J,Q (1,2,3,4,1)	0.50984
	2-S: QJ: 2,5,J,Q,A (1,2,3,4,1), shp (double-inside)	0.49923
	2-S: QJ: 2,7,8,J,Q (1,2,3,4,1), sp (double-inside)	0.50589
	2-S: QJ: 2,7,9,J,Q (1,2,3,4,1), sp (inside)	0.50194
	2-S: QJ: 2,7,10,J,Q (1,2,3,4,1), sp (outside)	0.49800
	2-S: QJ: 2,8,10,J,Q (1,2,3,4,1), 2sp (double-inside+outside)	0.49604
	2-S: QJ: 2,8,9,J,Q (1,2,3,4,1), 2sp (double-inside+inside)	0,49898
	2-S: QJ: 2,8,J,Q,A (1,2,3,4,1), shp (double-inside), sp (double-inside)	0.49528
	2-S: QJ: 2,9,J,Q,A (1,2,3,4,1), shp (double-inside), sp (inside)	0.49134
30	2-S: KJ if JT fp: 2,5,10,J,K (1,2,2,2,1)	0.48615
	2-S: KJ if JT fp: 2,9,10,J,K (1,2,1,1,3), sp	0.48319
31	2-RF: JT: 2,5,6,T,J (1,2,3,4,4)	0.51533
	2-RF: JT: 5,6,T,J,A (1,2,3,3,4), hp	0.50472
	2-RF: JT: 2,5,7,T,J (1,2,3,4,4), sp (doube-inside)	0.51138
	2-RF: JT: 2,5,8,T,J (1,2,3,4,4), sp (inside)	0.50743
	2-RF: JT: 2,5,9,T,J (1,2,3,4,4), sp (outside)	0.50348
	2-RF: JT: 2,7,T,J,K (1,2,3,3,4), shp (inside), sp (doube-inside)	0.49682
	2-RF: JT: 2,7,T,J,A (1,2,3,3,4), shp (double-inside), sp (doube-inside)	0.50077
	2-RF: JT: 2,8,T,J,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.49682
	2-RF: JT: 2,9,T,J,A (1,2,3,3,4), shp (double-inside), sp (outside)	0.49288
32	2-S: KH: 2,7,8,Q,K (1,2,3,4,1)	0.49405
	2-S: KH: 2,8,T,Q,K (1,2,3,4,1), sp (inside)	0.48615
	2-S: KH: 2,8,9,Q,K (1,2,3,4,1), sp (doube-inside)	0.49010
	2-S: KH: 2,9,T,Q,K (1,2,3,4,1), 2sp (double-inside+inside)	0.48319
33**	2-S: AQ if QT fp: 2,5,T,Q.A (1,2,2,2,3)	0.47431

Rank	Description	E[R]
34	2-RF: QT: 2,3,6,T,Q (1,2,2,3,3)	0.49707
	2-RF: QT: 2,3,T,Q,A (1,2,3,3,4), shp (double-inside)	0.48646
	2-RF: QT: 2,7,8,T,Q (1,2,2,3,3), sp (double-inside)	0,49312
	2-RF: QT: 2,7,9,T,Q (1,2,2,3,3), sp (inside)	0,48918
	2-RF: QT: 2,9,T,Q,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.47857
	2-RF: QT: 2,8,T,Q,A (1,2,2,3,3), shp (double-inside), sp (double-inside)	0.48252
	2-RF: QT: 8,9,T,Q,A (1,2,3,3,4), shp (double-inside), 2sp (double-inside+inside)	0.47561
	2-RF: QT: 2,8,9,T,Q (1,2,3,4,4), 2sp (double-inside+inside)	0.48622
35	2-S: AH: 2,5,8,K,A (1,2,3,4,1)	0.47826
	2-S: AH: 2,5,T,K,A (1,2,3,4,1), sp (double-inside)	0.47431
36**	1-RF: K if KT fp and 9p: 2,5,9,T,K (1,2,1,2,2)	0.45977
37**	2-RF: KT :2,5,8,T,K (1,1,2,3,3)	0.47882
	2-RF: KT :2,5,9,T,K (1,1,2,3,3), sp (double-inside)	0.47487
	2-RF: KT :2,5,8,T,K (1,2,3,3,3). fp	0.46216
38	J	
	1-RF: A,K,Q or J: 2,3,4,6,J (1,2,1,3,4)	0.48988
	1-RF: A,K,Q or J: 2,3,4,7,J (1,2,1,3,4), sp (double-inside)	0.48845
	1-RF: A,K,Q or J: 2,3,4,8,J (1,2,1,3,4), sp (inside)	0.48702
	1-RF: A,K,Q or J: 2,3,4,9,J (1,2,1,3,4), sp (outside)	0.48558
	1-RF: A,K,Q or J: 2,3,4,T,J (1,2,1,3,4), sp (outside.1)	0.48414
	1-RF: A,K,Q or J: 2,3,7,8,J (1,2,1,3,4), 2sp (double-inside+inside)	0.48594
	1-RF: A,K,Q or J: 2,3,7,9,J (1,2,1,3,4), 2sp (double-inside+outside)	0.48450
	1-RF: A,K,Q or J: 2,3,7,T,J (1,2,1,3,4), 2sp (double-inside+outside1)	0.48307
	1-RF: A,K,Q or J: 2,3,8,9,J (1,2,1,3,4), 2sp (inside+outside)	0.48342
	1-RF: A,K,Q or J: 2,3,8,T,J (1,2,1,3,4), 2sp (inside+outside1)	0.48199
	1-RF: A,K,Q or J: 2,3,9,T,J (1,2,1,3,4), 2sp (outside+outside1)	0.48091
	1-RF: A,K,Q or J: 2,7,8,9,J (1,2,1,3,4), 3sp (double-inside+inside+outside)	0.48262
	1-RF: A,K,Q or J: 2,7,8,T,J (1,2,1,3,4), 3sp (double-inside+inside+outiside1)	0.48118
	1-RF: A,K,Q or J: 2,7,9,T,J (1,2,1,3,4), 3sp (double-inside+outside+outside1)	0.48011
	1-RF: A,K,Q or J: 2,3,4,6,J (1,2,3,4,1), fp	0.47110
	1-RF: A,K,Q or J: 2,3,4,7,J (1,2,3,4,1), sp (double-inside), fp	0.48290
	1-RF: A,K,Q or J: 2,3,4,8,J (1,2,3,4,1), sp (inside), fp	0.48146
	1-RF: A,K,Q or J: 2,3,4,9,J (1,2,3,4,1), sp (outside), fp	0.48003
	1-RF: A,K,Q or J: 2,3,4,T,J (1,2,3,4,1), sp (outside1), fp	0.47859
	1-RF: A,K,Q or J: 2,3,7,8,J (1,2,3,4,1), 2sp (double-inside+inside), fp	0.48039
	1-RF: A,K,Q or J: 2,3,7,9,J (1,2,3,4,1), 2sp (double-inside+outside), fp	0.47895
	1-RF: A,K,Q or J: 2,3,7,T,J (1,2,3,4,1), 2sp (double-inside+outside1), fp	0.47751
	1-RF: A,K,Q or J: 2,3,8,9,J (1,2,3,4,1), 2sp (inside+outside), fp	0.47787
	1-RF: A,K,Q or J: 2,3,8,T,J (1,2,3,4,1), 2sp (inside+outside1), fp	0.47644
	1-RF: A,K,Q or J: 2,3,9,T,J (1,2,3,4,1), 2sp (outside+outside1), fp	0.47536
	1-RF: A,K,Q or J: 2,7,8,9,J (1,2,3,4,1), 3sp (double-inside+inside+outside), fp	0.47707
	1-RF: A,K,Q or J: 2,7,8,T,J (1,2,3,4,1), 3sp (double-inside+inside+outiside1), fp	0.46563
	1-RF: A,K,Q or J: 2,7,9,T,J (1,2,3,4,1), 3sp (double-inside+outside+outside1), fp	0.47455
	1-RF: A,K,Q or J: 2,3,4,6,J (1,1,2,3,4), fp	0.48030
	1-RF: A,K,Q or J: 2,3,4,7,J (1,1,2,3,1), sp (double-inside), fp	0.48030
	1-RF: A,K,Q or J: 2,3,4,7,J (1,2,3,1,1), sp (double-inside), fp	0.47864
	1-RF: A,K,Q or J: 2,3,4,8,J (1,1,2,3,1), sp (inside), fp	0.47743
	1-RF: A,K,Q or J: 2,3,4,8,J (1,2,3,1,1), sp (inside), fp	0.47698
	1-RF: A,K,Q or J: 2,3,4,9,J (1,1,2,3,1), sp (outside), fp	0.47759

Rank	Description	E[R]
38	1-RF: A,K,Q or J: 2,3,4,9,J (1,2,3,1,1), sp (outside), fp	0.47532
	1-RF: A,K,Q or J: 2,3,4,T,J (1,1,2,3,1), sp (outside.1), fp	0.47746
	1-RF: A,K,Q or J: 2,3,7,8,J (1,1,2,3,1), 2sp (double-inside+inside), fp	0.47635
	1-RF: A,K,Q or J: 2,3,7,8,J (1,2,1,3,1), 2sp (double-inside+inside), fp	0.47612
	1-RF: A,K,Q or J: 2,3,7,8,J (1,2,3,1,1), 2sp (double-inside+inside), fp	0.47590
	1-RF: A,K,Q or J: 2,3,7,9,J (1,1,2,3,1), 2sp (double-inside+outside), fp	0.47491
	1-RF: A,K,Q or J: 2,3,7,9,J (1,2,1,3,1), 2sp (double-inside+outside), fp	0.47469
	1-RF: A,K,Q or J: 2,3,7,9,J (1,2,3,1,1), 2sp (double-inside+outside), fp	0.47424
	1-RF: A,K,Q or J: 2,3,7,T,J (1,1,2,3,1), 2sp (double-inside+outside1), fp	0.47348
	1-RF: A,K,Q or J: 2,3,7,T,J (1,2,1,3,1), 2sp (double-inside+outside1), fp	0.47325
	1-RF: A,K,Q or J: 2,3,8,9,J (1,1,2,3,1), 2sp (inside+outside), fp	0.47384
	1-RF: A,K,Q or J: 2,3,8,9,J (1,2,1,3,1), 2sp (inside+outside), fp	0.47339
	1-RF: A,K,Q or J: 2,3,8,9,J (1,2,3,1,1), 2sp (inside+outside), fp	0.47316
	1-RF: A,K,Q or J: 2,3,8,T,J (1,1,2,3,1), 2sp (inside+outside1), fp	0.47240
	1-RF: A,K,Q or J: 2,3,8,T,J (1,2,1,3,1), 2sp (inside+outside1), fp	0.47195
	1-RF: A,K,Q or J: 2,3,9,T,J (1,1,2,3,1), 2sp (inside+outside1), fp	0.47133
	1-RF: A,K,Q or J: 2,3,9,T,J (1,2,1,3,1), 2sp (inside+outside1), fp	0.47065
	Q	
	1-RF: A,K,Q or J: 2,5,6,7,Q (1,2,1,3,4)	0.48392
	1-RF: A,K,Q or J: 2,5,6,8,Q (1,2,1,3,4), sp (double-inside)	0.48248
	1-RF: A,K,Q or J: 2,5,6,9,Q (1,2,1,3,4), sp (inside)	0.48105
	1-RF: A,K,Q or J: 2,5,6,T,Q (1,2,1,3,4), sp (outside)	0.47961
	1-RF: A,K,Q or J: 2,5,8,9,Q (1,2,1,3,4), 2sp (double-inside+inside)	0.47997
	1-RF: A,K,Q or J: 2,5,8,T,Q (1,2,1,3,4), 2sp (double-inside+inside)	0.47854
	1-RF: A,K,Q or J: 2,5,9,T,Q (1,2,1,3,4), 2sp (inside+outside)	0.47746
	1-RF: A,K,Q or J: 2,8,9,T,Q (1,2,1,3,4), 3sp (double-inside+inside+outside)	0.47665
	1-RF: A,K,Q or J: 2,5,6,7,Q (1,2,3,4,1), fp	0.47867
	1-RF: A,K,Q or J: 2,5,6,8,Q (1,2,3,4,1), sp (double-inside), fp	0.47693
	1-RF: A,K,Q or J: 2,5,6,9,Q (1,2,3,4,1), sp (inside), fp	0.47550
	1-RF: A,K,Q or J: 2,5,6,T,Q (1,2,3,4,1), sp (outside), fp	0.47406
	1-RF: A,K,Q or J: 2,5,8,9,Q (1,2,3,4,1), 2sp (double-inside+inside), fp	0.47442
	1-RF: A,K,Q or J: 2,5,8,T,Q (1,2,3,4,1), 2sp (double-inside+inside), fp	0.47299
	1-RF: A,K,Q or J: 2,5,9,T,Q (1,2,3,4,1), 2sp (inside+outside), fp	0.47191
	1-RF: A,K,Q or J: 2,8,9,T,Q (1,2,3,4,1), 3sp (double-inside+inside+outside), fp	0.47110
	1-RF: A,K,Q or J: 2,5,6,7,Q (1,1,2,3,1), fp	0.47433
	1-RF: A,K,Q or J: 2,5,6,8,Q (1,1,2,3,1), sp (double-inside), fp	0.47290
	1-RF: A,K,Q or J: 2,5,6,8,Q (1,2,3,1,1), sp (double-inside), fp	0.47267
	1-RF: A,K,Q or J: 2,5,6,9,Q (1,1,2,3,1), sp (inside), fp	0.47146
	1-RF: A,K,Q or J: 2,5,6,9,Q (1,2,3,1,1), sp (inside), fp	0.47101
	1-RF: A,K,Q or J: 2,5,6,T,Q (1,1,2,3,1), sp (outside), fp	0.47002
	1-RF: A,K,Q or J: 2,5,6,T,Q (1,2,3,1,1), sp (outside), fp	0.48042
	1-RF: A,K,Q or J: 2,5,8,9,Q (1,1,2,3,1), 2sp (double-inside+inside), fp	0.47038
	1-RF: A,K,Q or J: 2,5,8,9,Q (1,2,1,3,1), 2sp (double-inside+inside), fp	0.47016
	1-RF: A,K,Q or J: 2,5,8,9,Q (1,2,3,1,1), 2sp (double-inside+inside), fp	0.46994
	1-RF: A,K,Q or J: 2,5,8,T,Q (1,1,2,3,1), 2sp (double-inside+outside), fp	0.46895
	1-RF: A,K,Q or J: 2,5,8,T,Q (1,2,1,3,1), 2sp (double-inside+outside), fp	0.46872
	1-RF: A,K,Q or J: 2,5,9,T,Q (1,1,2,3,1), 2sp (inside+outside), fp	0.46787
	1-RF: A,K,Q or J: 2,5,9,T,Q (1,2,1,3,1), 2sp (inside+outside), fp	0.46742
	1-RF: A,K,Q or J: 2,8,9,T,Q (1,1,2,3,1), 3sp (double-inside+inside+outside), fp	0.46684
	1-RF: A,K,Q or J: 2,8,9,T,Q (1,2,1,3,1), 3sp (double-inside+inside+outside), fp	0.46662

Rank	Description	**E**[**R**]
38	K	
	1-RF: A,K,Q or J: 2,5,7,8,K (1,2,1,3,4)	0.47795
	1-RF: A,K,Q or J: 2,5,7,9,K (1,2,1,3,4), sp (double-inside)	0.47652
	1-RF: A,K,Q or J: 2,5,7,T,K (1,2,1,3,4), sp (inside)	0.47508
	1-RF: A,K,Q or J: 2,5,9,T,K (1,2,1,3,4), 2sp (double-inside+inside)	0.47401
	1-RF: A,K,Q or J: 2,5,7,8,K (1,2,3,4,1), fp	0.47240
	1-RF: A,K,Q or J: 2,5,7,9,K (1,2,3,4,1), sp (double-inside), fp	0.47067
	1-RF: A,K,Q or J: 2,5,7,T,K (1,2,3,4,1), sp (inside), fp	0.46953
	1-RF: A,K,Q or J: 2,5,9,T,K (1,2,3,4,1), 2sp (double-inside+inside), fp	0.46846
	1-RF: A,K,Q or J: 2,5,7,8,K (1,1,2,3,1), fp	0.46837
	1-RF: A,K,Q or J: 2,5,7,9,K (1,1,2,3,1), sp (double-inside), fp	0.46693
	1-RF: A,K,Q or J: 2,5,7,9,K (1,2,3,1,1), sp (double-inside), fp	0.46671
	1-RF: A,K,Q or J: 2,5,7,T,K (1,1,2,3,1), sp (inside), fp	0.46549
	1-RF: A,K,Q or J: 2,5,9,T,K (1,1,2,3,1), 2sp (double-inside+inside), fp	0.46442
	1-RF: A,K,Q or J: 2,5,9,T,K (1,2,1.3,1), 2sp (double-inside+inside), fp	0.46419
	A	
	1-RF: A,K,Q or J: 2,6,7,8,A (1,2,1,3,4), sp	0.47652
	1-RF: A,K,Q or J: 6,7,8,T,A (1,2,1,3,4), sp	0.47652
	1-RF: A,K,Q or J: 2,3,7,8,A (1,2,1,3,4), 2sp	0.47544
	1-RF: A,K,Q or J: 2,6,7,T,A (1,2,1,3,4), 2sp	0.47508
	1-RF: A,K,Q or J: 2,3,7,8,A (1,2,1,3,4), 3sp	0.47463
	1-RF: A,K,Q or J: 2,5,7,T,A (1,2,1,3,4), 3sp	0.47401
	1-RF: A,K,Q or J: 2,3,4,T,A (1,2,1,3,4), 4sp	0.47320
	1-RF: A,K,Q or J: 2,6,7,8,A (1,2,3,4,1), sp, fp	0.47074
	1-RF: A,K,Q or J: 6,7,8,T,A (1,2,3,4,1), sp, fp	0.47097
	1-RF: A,K,Q or J: 6,7,8,T,A (1,2,3,4,4), sp, fp	0.46654
	1-RF: A,K,Q or J: 2,3,7,8,A (1,2,3,4,1), 2sp, fp	0.46967
	1-RF: A,K,Q or J: 2,6,7,T,A (1,2,3,4,1), 2sp, fp	0.46931
	1-RF: A,K,Q or J: 2,6,7,T,A (1,2,3,4,4), 2sp, fp	0.46510
	1-RF: A,K,Q or J: 2,5,7,T,A (1,2,3,4,1), 3sp, fp	0.46823
	1-RF: A,K,Q or J: 2,5,7,T,A (1,2,3,4,4), 3sp, fp	0.46403
	1-RF: A,K,Q or J: 2,3,4,T,A (1,2,3,4,1), 4sp, fp	0.46742
	1-RF: A,K,Q or J: 2,3,4,T,A (1,2,3,4,4), 4sp, fp	0.46322
39**	3-SF: $s + h = 1$: 2,3,5,7,9 (1,2,3,3,3)	0.44311

Appendix B

Double Bonus

Rank	Description	E[R]
1**	5-RF: T,J,Q,K,A (1,1,1,1,1)	800.00000
2**	5-4K: 2,A,A,A,A (1,1,2,3,4)	160.00000
	5-4K: 2,3,3,3,3 (1,1,2,3,4)	80.00000
	5-4K: 2,8,8,8,8 (1,1,2,3,4)	50.00000
3**	5-SF: 2,3,4,5,6 (1,1,1,1,1)	50.00000
4	4-RF: *,J,Q,K,A (1,1,1,1,1)	18.63830
	4-RF: *,J,Q,K,A (1,2,2,2,2)	18.78723 (2-9)
		18.68085 (T), 18.76596 (J-A)
	4-RF: *,T,Q,K,A (1,1,1,1,1)	18.57447
	4-RF: *,T,Q,K,A (1,2,2,2,2)	18.72340 (2-T)
		18.61702 (J), 18.70213 (Q-A)
	4-RF: *,T,J,K,A (1,1,1,1,1)	18.57447
	4-RF: *,T,J,K,A (1,2,2,2,2)	18.72340 (2-T)
		18.70213 (J,K,A), 18.61702 (Q)
	4-RF: *,T,J,Q,A (1,1,1,1,1)	18.57447
	4-RF: *,T,J,Q,A (1,2,2,2,2)	18.7234 (2-T)
		18.70213 (J,Q,A), 18,61702 (K)
	4-RF: *,T,J,Q,K (1,1,1,1,1)	19.80851
	4-RF: *,T,J,Q,K (1,2,2,2,2)	19.95744 (2-8,T)
		19.85106 (9,A), 19.93617 (J-K)
5	3-3K with aces: 2,T,A,A,A (1,2,1,2,3)	10.10823
	3-3K with aces: 2,2,A,A,A (1,2,1,2,3), 5-FH	10.11471
6**	5-FH: 2,2,T,T,T, (1,2,1,2,3)	10.00000
7**	5-F: 2,4,5,7,T (1,1,1,1,1)	7.00000
8	3-K with 2s, 3s, 4s: 2,2,2,4,5, (1,2,3,1,2)	6.70398
	3-K with others: 5,5,5,3,7, (1,2,3,1,2)	5.42738
9**	5-S: 2,3,4,5,6 (1,2,3,4,1)	5.00000

Rank	Description	E[R]
10	4-SF (outside): 2,3,4,5,T (1,1,1,1,2), 0H	3.80851
	4-SF (outside): 8,9,T,J,K (1,1,1,1,2), 1H	3.87234
	4-SF (outside): 8,9,T,J,J (1,1,1,1,2), 1H, 2-HP	3.85106
	4-SF (outside): 9,T,J,Q,A (1,1,1,1,2), 2H	3.93617
	4-SF (outside): 9,T,J,Q,Q (1,1,1,1,2), 2H, 2-HP	3.91489
	4-SF (inside): 2,3,4,6,8 (1,1,1,1,2), 0H	2.57447
	4-SF (inside): 7,8,10,J,A (1,1,1,1,2), 1H	2.63830
	4-SF (inside): 7,8,10,J,J (1,1,1,1,2), 1H, 2-HP	2.61702
	4-SF (inside): 8,9,J,Q,A (1,1,1,1,2), 2H	2.70213
	4-SF (inside): 8,9,J,Q,Q (1,1,1,1,2), 2H, 2-HP	2.68085
	4-SF (inside): 7,9,J,Q,K (1,2,2,2,2), 3H	2.76596
	4-SF (inside): 9,J,Q,K,K (1,1,1,1,2), 3H, 2-HP	2.74468
11**	4-2P: 2,2,A,A,5 (1,2,1,2,1)	1.76596
12**	2-HP with As: A,A,5,7,T (1,2,1,2,1)	1.76152
13	3-RF: QJT: 3,7,T,J,Q (1,1,2,2,2)	1.58464
	3-RF: QJT: 3,T,J,Q,A (1,2,2,2,1), shp (double-inside)	1.56337
	3-RF: QJT: 3,T,J,Q,K (1,2,2,2,1), shp (outside)	1.54487
	3-RF: QJT: 3,8,T,J,Q (1,1,2,2,2), sp (inside)	1.56614
	3-RF: QJT: 3,9,T,J,Q (1,1,2,2,2), sp (outside)	1.54764
	3-RF: QJT: 3,7,T,J,Q (1,2,2,2,2), fp	1.52636
	3-RF: QJT: 7,7,T,J,Q (1,2,3,3,3), 2-LP (2-7)	1.58464
	3-RF: QJT: 7,T,T,J,Q (1,2,3,3,3), 2-LP (T)	1.57909
	3-RF: QJT: 8,T,J,Q,A (1,2,2,2,3), shp (inside), sp (inside)	1.54487
	3-RF: QJT: 8,T,J,Q,K (1,2,2,2,3), shp (outside), sp (inside)	1.52636
	3-RF: QJT: 9,T,J,Q,A (1,2,2,2,3), shp (inside), sp (outside)	1.52636
	3-RF: QJT: 3,T,J,Q,A (1,1,1,1,2), shp (inside), fp	1.50508
	3-RF: QJT: 3,T,J,Q,K (1,1,1,1,2), shp (outside), fp	1.48659
	3-RF: QJT: T,T,J,Q,A (1,2,2,2,3), shp (inside), 2-LP (T)	1.55782
	3-RF: QJT: T,T,J,Q,K (1,2,2,2,3), shp (outside), 2-LP (T)	1.53932
	3-RF: QJT: 3,8,T,J,Q (1,2,1,1,1), sp (inside), fp	1.50786
	3-RF: QJT: 3,9,T,J,Q (1,2,1,1,1), sp (outside), fp	1.48936
	3-RF: QJT: 8,8,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (8)	1.54764
	3-RF: QJT: 9,9,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (9)	1.51064
	3-RF: QJT: 8,T,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (T)	1.56059
	3-RF: QJT: 9,T,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (T)	1.54209
	3-RF: QJT: 7,7,T,J,Q (1,2,1,1,1), fp, 2-LP (2-7)	1.52636
	3-RF: QJT: 7,T,T,J,Q (1,1,2,1,1), fp, 2-LP (T)	1.52081
14	4-F: $KQJx$: 3,8,J,Q,K (1,2,2,2,2), fp	1.53191
	4-F: $KQJx$: 3,J,Q,K,A (1,1,1,1,2), shp (inside), fp	1.53191
	4-F: $KQJx$: 3,9,J,Q,K (1,2,1,1,1), sp (inside), fp	1.53191
	4-F: $KQJx$: 3,T,J,Q,K (1,2,1,1,1), sp (outside), fp	1.53191
	4-F: $KQJx$: 8,8,J,Q,K (1,2,1,1,1), fp, 2-LP (2-8)	1.53191
	4-F: $KQJx$: 3,J,Q,K,K (1,1,1,1,2), fp, 2-HP	1.51064

Rank	**Description**	**E[R]**
15	3-RF: *KQJ*: 3,8,J,Q,K (1,1,2,2,2)	1.57539
	3-RF: *KQJ*: 3,J,Q,K,A (1,2,2,2.3), shp (inside)	1.55412
	3-RF: *KQJ*: 3,9,J,Q,K (1,1,2,2,2), sp (inside)	1.55689
	3-RF: *KQJ*: 3,T,J,Q,K (1,1,2,2,2), sp (outside)	1.53839
	3-RF: *KQJ*: 8,8,J,Q,K (1,2,3,3,3), 2-LP (2-8)	1.57539
	3-RF: *KQJ*: 3,J,Q,K,K (1,2,2,2,3), 2-HP	1.53654
	3-RF: *KQJ*: 9,J,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.53562
	3-RF: *KQJ*: 9,9,J,Q,K (1,2,3,3,3), sp (inside), 2-LP	1.53839
	3-RF: *KQJ*: T,T,J,Q,K (1,2,3,3,3), sp (outside), 2-LP	1.50139
	3-RF: *KQJ*: 9,J,Q,K,K (1,2,2,2,3), sp (inside), 2-HP	1.51804
	3-RF: *KQJ*: T,J,Q,K,K (1,2,2,2,3), sp (outside), 2-HP	1.49954
	3-RF: *KQJ*: J,Q,K,K,A (1,2,2,3,4), shp (inside), sp (outside), 2-HP	1.51526
16**	2-HP with Js, Qs, Ks: 2,3,6,K,K (1,2,3,4,1)	1.45624
17**	*AHTx*+K,Q,J or T: 9,T,Q,K.A (1.1,2,1,1)	1.46809
	AHTx+K,Q,J or T: 9,T,Q,K.A (1.1,1,2,1)	1.46809
	AHTx+K,Q,J or T: 9,T,J,K.A (1.1,1,2,1)	1.46809
18	4-F: 4,7,9,J,Q (1,2,2,2,2), 2H	1.46809
	4-F: 4,7,9,T,K (1,2,2,2,2), 1H	1.40426
	4-F: 3,4,7,8,9 (1,1,1,2,1), 0H	1.34043
19	*KQT*	
	3-RF: *KQT*: 3,8,T,Q,K (1,1,2,2,2)	1.47549
	3-RF: *KQT*: 3,T,Q,K,A (1,2,2,2,3), shp (inside)	1.45421
	3-RF: *KQT*: 3,T,J,Q,K (1,2,3,2,2), shp (outisde)	1.43571
	3-RF: *KQT*: 3,9,T,Q,K (1,1,2,2,2), sp (inside)	1.45698
	3-RF: *KQT*: 8,8,T,Q,K (1,2,3,3,3), 2-LP (2-8)	1.47549
	3-RF: *KQT*: 8,T,T,Q,K (1,2,3,3,3), 2-LP (T)	1.46994
	3-RF: *KQT*: 9,T,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.43571
	3-RF: *KQT*: T,T,Q,K,A (1,2,2,2,3), shp (inside), 2-LP (T)	1.44866
	3-RF: *KQT*: T,T,J,Q,K (1,2,2,2,3), shp (outisde), 2-LP (T)	1.43016
	3-RF: *KQT*: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.43848
	3-RF: *KQT*: 9,T,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (T)	1.45143
	3-RF: *KQT*: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP	1.43848
	KJT	
	3-RF: *KJT*: 3,8,T,J,K (1,1,2,2,2)	1.47549
	3-RF: *KJT*: 3,T,J,K,A (1,2,2,2,3), shp (inside)	1.45421
	3-RF: *KJT*: 8,T,J,Q,K (1,2,2,3,2), shp (outside)	1.43571
	3-RF: *KJT*: 3,9,T,J,K (1,1,2,2,2), sp (inside)	1.45698
	3-RF: *KJT*: 8,8,T,J,K (1,2,3,3,3), 2-LP (2-8)	1.47549
	3-RF: *KJT*: 8,T,T,J,K (1,2,3,3,3), 2-LP (T)	1.46994
	3-RF: *KJT*: 9,T,J,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.43571
	3-RF: *KJT*: T,T,J,K,A (1,2,3,3,3), shp (inside), 2-LP (T)	1.44866
	3-RF: *KJT*: T,T,J,Q,K (1,2,2,3,2), shp (outside), 2-LP (T)	1.43016
	3-RF: *KJT*: 9,9,T,J,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.43848
	3-RF: *KJT*: 9,T,T,J,K (1,2,3,3,3), sp (inside), 2-LP (T)	1.45143

Rank	Description	E[R]
19	*AKQ*	
	3-RF: *AKQ*: 8,9,Q,K,A (1,1,2,2,2)	1.46623
	3-RF: *AKQ*: 8,J,Q,K,A (1,1,2,2,2), shp (inside)	1.44496
	3-RF: *AKQ*: 8,T,Q,K,A (1,1,2,2,2), sp (inside)	1.44773
	3-RF: *AKQ*: 9,9,Q,K,A (1,2,2,2,2), 2-LP (2-9)	1.46623
	3-RF: *AKQ*: T,T,Q,K,A (1,2,3,3,3), sp (inside), 2-LP (T)	1.42923
	AKJ	
	3-RF: *AKJ*: 8,9,J,K,A (1,1,2,2,2)	1.46623
	3-RF: *AKJ*: 8,J,Q,K,A (1,2,3,3,3), shp (inside)	1.44496
	3-RF: *AKJ*: 3,T,J,K,A (1,2,3,3,3), sp (inside)	1.44773
	3-RF: *AKJ*: 9,9,J,K,A (1,1,2,2,2), 2-LP (2-9)	1.46623
	3-RF: *AKJ*: T,T,J,K,A (1,2,3,3,3), sp (inside), 2-LP	1.42923
	AQJ	
	3-RF: *AQJ*: 3,8,J,Q,A (1,1,2,2,2)	1.46623
	3-RF: *AQJ*: 8,J,Q,K,A (1,2,2,3,2), shp (inside)	1.44496
	3-RF: *AQJ*: 3,T,J,Q,A (1,2,3,3,3), sp (inside)	1.44773
	3-RF: *AQJ*: 9,9,J,Q,A (1,2,3,3,3), 2-LP (2-9)	1.46623
	3-RF: *AQJ*: T,T,J,Q,A (1,2,3,3,3), sp (inside), 2-LP	1.42923
	AKT	
	3-RF: *AKT*: 8,9,T,K,A (1,1,2,2,2)	1.36633
	3-RF: *AKT*: 8,T,Q,K,A (1,2,3,2,2), shp (inside)	1.34505
	3-RF: *AKT*: 8,T,J,K,A (1,2,3,2,2), shp (inside)	1.34505
	3-RF: *AKT*: 9,9,T,K,A (1,2,3,3,3), 2-LP (2-9)	1.36633
	3-RF: *AKT*: 9,T,T,K,A (1,2,3,3,3), 2-LP (T)	1.36078
	3-RF: *AKT*: T,T,Q,K,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.33950
	3-RF: *AKT*: T,T,J,K,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.33950
	AQT	
	3-RF: *AQT*: 3,9,T,Q,A (1,1,2,2,2)	1.36633
	3-RF: *AQT*: 8,T,Q,K,A (1,2,2,3,2), shp (inside)	1.34505
	3-RF: *AQT*: 8,T,J,Q,A (1,2,3,2,2), shp (inside)	1.34505
	3-RF: *AQT*: 9,9,T,Q,A (1,2,3,3,3), 2-LP (2-9)	1.36633
	3-RF: *AQT*: 9,T,T,Q,A (1,2,3,3,3), 2-LP (T)	1.36078
	3-RF: *AQT*: T,T,Q,K,A (1,2,1,3,1), shp (inside), 2-LP (T)	1.33950
	3-RF: *AQT*: T,T,J,Q,A (1,2,3,1,1), shp (inside), 2-LP (T)	1.33950
	AJT	
	3-RF: *AJT*: 3,9,T,J,A (1,1,2,2,2)	1.36633
	3-RF: *AJT*: 3,T,J,K,A (1,2,2,3,2), shp (inside)	1.34505
	3-RF: *AJT*: 3,T,J,Q,A (1,2,2,3,2), shp (inside)	1.34505
	3-RF: *AJT*: 9,9,T,J,A (1,2,3,3,3), 2-LP (2-9)	1.36633
	3-RF: *AJT*: 3,T,T,J,A (1,1,2,2,2), 2-LP (T)	1.36078
	3-RF: *AJT*: T,T,J,K,A (1,2,2,3,2), shp (inside), 2-LP (T)	1.33950
	3-RF: *AJT*: T,T,J,Q,A (1,2,2,3,2), shp (inside), 2-LP (T)	1.33950
20**	4-S: KQJT: 3,T,J,Q,K (1,2,3,4,1)	1.04255
21	4-S: 5432–QJT9: 2,9,T,J,Q (1,2,3,4,1), 2H QJT9	0.97872
	4-S: 5432–QJT9: 2,8,9,T,J (1,2,3,4,1), 1H JT98	0.91489
	4-S: 5432–QJT9: 2,3,4,5,T (1,2,3,4,1), 0H 5432-T987	0.85106
22**	2-LP: 2s, 3s, 4s: 2,2,5,10,13 (1,2,1,2,3)	0.82664

Rank	Description	E[R]
23	2H, 2S (inside)	
	3-SF: $s + h > 3$: 4,5,9,J,Q (1,2,3,3,3)	0.78168
	3-SF: $s + h > 3$: 4,9,J,Q,K (1,2,2,2,3), shp (inside)	0.76041
	3-SF: $s + h > 3$: 4,9,J,Q,A (1,2,2,2,3), hp	0.77891
	3-SF: $s + h > 3$: 4,8,9,J,Q (1,2,3,3,3), sp (inside)	0.76318
	3-SF: $s + h > 3$: 5,5,9,J,Q (1,2,3,3,3), 2-LP	0.78168
	3-SF: $s + h > 3$: 9,J,Q,K,A (1,1,1,2,3), shp (inside), hp	0.75763
	3-SF: $s + h > 3$: 8,9,J,Q,K (1,2,2,2,3), shp (inside), sp (inside)	0.74191
	3-SF: $s + h > 3$: 8,9,J,Q,A (1,2,2,2,3), hp, sp (inside)	0.76041
	1H, 3S (outside)	
	3-SF: $s + h > 3$: 4,5,9,10,J (1,2,3,3,3)	0.79094
	3-SF: $s + h > 3$: 4,9,10,J,K (1,2,2,2,3), shp (inside)	0.76966
	3-SF: $s + h > 3$: 5,9,10,J,A (1,2,2,2,3), hp	0.78166
	3-SF: $s + h > 3$: 4,7,9,10,J (1,2,3,3,3), sp (inside)	0.77243
	3-SF: $s + h > 3$: 9,10,J,K,A (1,1,1,2,3), shp (inside), hp	0.76688
	3-SF: $s + h > 3$: 7,9,10,J,K (1,2,2,2,3), shp (inside), sp (inside)	0.75166
	3-SF: $s + h > 3$: 7,9,10,J,A (1,2,2,2,3), hp, sp (inside)	0.76966
24**	2-LP: 5s–Ts: 5,5,8,10,13 (1,2,1,2,3) 5-10	0.74338
25**	4-S: AKQJ: 4,J,Q,K,A (1,2,3,4,1)	0.68085
26	0H, 3S (outside)	
	3-SF: $s + h = 3$: 4,5,8,9,10 (1,2,3,3,3)	0.69103
	3-SF: $s + h = 3$: 4,8,9,10,Q (1,2,2,2,3), shp (inside)	0.66975
	3-SF: $s + h = 3$: 4,8,9,10,K (1,2,2,2,3), hp	0.68825
	3-SF: $s + h = 3$: 4,6,8,9,10 (1,2,3,3,3), sp (inside)	0.67253
	3-SF: $s + h = 3$: 8,9,10,Q,A (1,1,1,2,3), shp (inside) hp	0.66698
	3-SF: $s + h = 3$: 6,8,9,10,Q (1,2,2,2,3), shp (inside), sp (inside)	0.65125
	3-SF: $s + h = 3$: 6,8,9,10,K (1,2,2,2,3), hp, sp (inside)	0.66975
	1H, 2S (inside)	
	3-SF: $s + h = 3$: 4,5,9,10,Q (1,2,3,3,3)	0.68178
	3-SF: $s + h = 3$: 4,9,10,Q,K (1,2,2,2,3), shp (inside)	0.66050
	3-SF: $s + h = 3$: 4,9,10,Q,A (1,2,2,2,3), hp	0.67900
	3-SF: $s + h = 3$: 4,8,9,10,Q (1,2,3,3,3), sp (inside)	0,66327
	3-SF: $s + h = 3$: 9,10,Q,K,A (1,1,1,2,3), shp (inside), hp	0.65772
	3-SF: $s + h = 3$: 8,9,10,Q,K (1,2,2,2,3), shp (inside), sp (inside)	0.64200
	3-SF: $s + h = 3$: 8,9,10,Q,A (1,2,2,2,3), hp, sp (inside)	0.64200
	2H, 1S (double-inside)	
	3-SF: $s + h = 3$: 4,5,9,J,K (1,2,3,3,3)	0.67253
	3-SF: $s + h = 3$: 4,9,J,Q,K (1,2,2,3,2), shp (inside)	0.65125
	3-SF: $s + h = 3$: 4,9,J,K,A (1,2,2,2,3), hp	0.66975
	3-SF: $s + h = 3$: 4,9,T,J,K (1,2,3,2,2), sp (inside)	0.65402
	3-SF: $s + h = 3$: 9,T,J,K,A (1,2,1,1,3), hp, sp (inside)	0.65125
27**	4-S: AHHT or KQJ9: 2,T,Q,K,A (1,1,2,3,4)	0.61702
	4-S: AHHT or KQJ9: 7,9,J,Q,K (1,1,2,3,4)	0.61702

Rank	Description	E[R]
28	2-RF: QJ: 2,5,7,J,Q (1,2,3,4,4)	0.60654
	2-RF: QJ: 6,7,J,Q,A (1,2,3,3,4), shp (double-inside)	0.59494
	2-RF: QJ: 6,7,J,Q,K (1,2,3,3,4), shp (inside)	0.59001
	2-RF: QJ: 6,7,8,J,Q,(1,2,3,4,4), sp (double-inside)	0.60160
	2-RF: QJ: 6,7,9,J,Q,(1,2,3,4,4), sp (inside)	0.59700
	2-RF: QJ: 6,7,T,J,Q,(1,2,3,4,4), sp (outside)	0.59174
	2-RF: QJ: 6,8,T,J,Q,(1,2,3,4,4), 2sp (double-inside+outside)	0.58804
	2-RF: QJ: 6,8,9,J,Q,(1,2,3,4,4), 2sp (double-inside+inside)	0.59297
	2-RF: QJ: 2,5,7,J,Q (1,2,3,3,3), fp	0.58711
	2-RF: QJ: 6,8,J,Q,A (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.59001
	2-RF: QJ: 6,9,J,Q,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.58058
	2-RF: QJ: 6,8,J,Q,K (1,2,3,3,4), shp (inside), sp (double-inside)	0.58058
	2-RF: QJ: 8,9,J,Q,A (1,2,3,3,4), shp (double-inside), 2sp (double-inside+inside)	0.58058
	2-RF: QJ: 6,7,J,Q,A (1,3,3,3,4), shp (double-inside), fp	0.57552
	2-RF: QJ: 6,7,J,Q,K (1,3,3,3,4), shp (inside), fp	0.57058
	2-RF: QJ: 6,8,J,Q,A (1,2,1,1,3), shp (double-inside), sp (double-inside), fp	0.56565
	2-RF: QJ: 6,9,J,Q,A (1,2,1,1,3), shp (double-inside), sp (inside), fp	0.56565
	2-RF: QJ: 6,8,J,Q,K (1,2,1,1,3), shp (inside), sp (double-inside), fp	0.56565
29	3-F: HHx: 2,7,8,J,K (1,1,2,2,2)	0.58446
	3-F: HHx: 2,7,J,K,A (1,2,2,2,3), shp (double-inside)	0.57286
	3-F: HHx: 2,7,J,Q,K (1,2,2,3,2), shp (inside)	0.56793
	3-F: HHx: 2,7,T,J,K (1,2,3,2,2), shp (inside)	0.57459
	3-F: HHx: 2,7,9,J,K (1,2,3,2,2), sp (double-nside)	0.57953
30	2-RF: KH: 5,7,8,Q,K (1,2,3,4,4)	0.58446
	2-RF: KH: 5,7,Q,K,A (1,2,3,3,4), shp (double-inside)	0.57286
	2-RF: KH: 5,7,J,Q,K (1,2,3,4,4), shp (inside)	0.56793
	2-RF: KH: 5,8,T,Q,K (1,2,3,4,4), sp (inside)	0.57459
	2-RF: KH: 5,8,9,Q,K (1,2,3,4,4), sp (double-inside)	0.57953
	2-RF: KH: 5,9,T,Q,K (1,2,3,4,4), 2sp (double-inside+inside)	0.57089
	2-RF: KH: 5,7,8,Q,K (1,2,3,3,3), fp	0.56503
	2-RF: KH: 5,9,Q,K,A (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.56793
31	3-SF: $s + h = 2$, no sp: 2,8,9,T,K (1,1,2,2,2), 1H, 1S	0.57262
	3-SF: $s + h = 2$, no sp: 2,9,T,K,A (1,2,2,2,3), hp	0.56984
	3-SF: $s + h = 2$, no sp: 7,8,J,K,A (1,1,1,2,3), 2hp	0.56707
	3-SF: $s + h = 2$, no sp: 2,3,6,8,9 (1,1,2,2,2), 0H, 2S	0.58187
	3-SF: $s + h = 2$, no sp: 2,6,8,9,K (1,2,2,2,3), hp	0.57909
	3-SF: $s + h = 2$, no sp: 6,8,9,K,A (1,1,1,2,3), 2hp	0.57632
32	2-RF: AH: 5,8,9,K,A (1,2,3,4,4)	0.56916
	2-RF: AH: 5,8,Q,K,A (1,2,3,4,4), shp (inside)	0.55757
	2-RF: AH: 5,8,T,K,A (1,2,3,4,4), sp (double-inside)	0.56423
33	3-SF: $s + h = 2$, sp: 2,8,9,T,Q (1,2,2,3,2), 1H, 1S, sp (inside)	0.55412
	3-SF: $s + h = 2$, sp: 2,6,8,9,T (1,2,2,2,3), 0H, 2S, sp (inside)	0.56337
	3-SF: $s + h = 2$, sp: 2,7,8,T,J (1,2,2,2,3), shp (inside)	0.56059
	3-SF: $s + h = 2$, sp: 7,8,T,J,A (1,1,1,2,3), shp (inside), hp	0.55782
	3-SF: $s + h = 2$, sp: 6,7,8,T,J (1,2,2,2,3), shp (inside), sp (inside)	0,54209
	3-SF: $s + h = 2$, sp: 5,6,8,9,A (1,2,2,2,3), sp (inside), hp	0.56059
	3-SF: $s + h = 2$, sp: 6,8,9,K,A (1,1,1,2,3), 2hp	0.57632
34**	4-S with 2H: 2,8,T,J,Q (1,2,3,4,1), 2H	0.55319
35**	3-S: KQJ: 2,5,J,Q,K (1,1,2,3,4)	0.51989

Rank	Description	E[R]
36**	3-S: QJT: 2,5,T,J,Q (1,2,3,4,1)	0.49398
37**	4-S with 1H: 2,8,9,T,Q (1,2,3,4,1), 1H	0.48936
38	2-RF: JT: 2,5,6,T,J (1,2,3,4,4)	0.50114
	2-RF: JT: 5,6,T,J,A (1,2,3,3,4), shp (double-inside)	0.48955
	2-RF: JT: 2,5,7,T,J (1,2,3,4,4), sp (doube-inside)	0.49621
	2-RF: JT: 2,5,8,T,J (1,2,3,4,4), sp (inside)	0.49127
	2-RF: JT: 2,5,9,T,J (1,2,3,4,4), sp (outside)	0.48634
	2-RF: JT: 2,7,T,J,K (1,2,3,3,4), shp (inside), sp (doube-inside)	0.47968
	2-RF: JT: 2,7,T,J,A (1,2,3,3,4), shp (double-inside), sp (doube-inside)	0.48461
	2-RF: JT: 2,8,T,J,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.47968
	2-RF: JT: 2,9,T,J,A (1,2,3,3,4), shp (double-inside), sp (outside)	0,47475
39	2-S: QJ: 2,5,6,J,Q (1,2,3,4,1)	0.48202
	2-S: QJ: 2,5,J,Q,A (1,2,3,4,1), shp (inside)	0.47043
	2-S: QJ: 2,7,8,J,Q (1,2,3,4,1), sp (double-inside)	0.47709
	2-S: QJ: 2,7,9,J,Q (1,2,3,4,1), sp (inside)	0.47216
	2-S: QJ: 2,8,J,Q,A (1,2,3,4,1), shp (inside), sp (double-inside)	0.46549
	2-S: QJ: 2,9,J,Q,A (1,2,3,4,1), shp (inside), sp (inside)	0.46051
40	3-SF: $s + h = 1$: 2,3,5,7,9 (1,2,3,3,3)	0.47271
	3-SF: $s + h = 1$: 2,5,7,9,A (1,2,2,2,3), hp	0.46994
41	3-F: Hxx : 2,5,8,T,K (1,2,3,3,3)	0.46346
	3-F: Hxx : 2,8,T,J,K (1,2,2,3,2), shp (inside)	0,46068
	3-F: Hxx : 2,8,T,K,A (1,2,2,2,3), hp	0,46068
	3-F: Hxx : 2,5,9,T,K (1,2,3,2,2), sp (inside)	0.46346
42	2-RF: QT : 2,3,6,T,Q (1,2,2,3,3)	0.47906
	2-RF: QT : 2,3,T,Q,A (1,2,3,3,4), shp (double-inside)	0.46747
	2-RF: QT : 2,3,T,Q,K (1,2,3,3,4), shp (inside)	0.46253
	2-RF: QT : 2,7,8,T,Q (1,2,2,3,3), sp (double-inside)	0.47413
	2-RF: QT : 2,7,9,T,Q (1,2,2,3,3), sp (inside)	0.46920
	2-RF: QT : 2,8,T,Q,A (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.46253
	2-RF: QT : 2,9,T,Q,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.45760
	2-RF: QT : 2,8,T,Q,K (1,2,3,3,4), shp (double-inside), sp (double-inside)	0.45760
43**	3-S: AHH if 9p: 2,9,J,K,A (1,2,3,4,1)	0.44588
44**	2-S: AH, no sp: 2,5,8,K,A (1,2,3,4,1)	0.44934
45	2-S: KH: 2,7,8,Q,K (1,2,3,4,1)	0.46229
	2-S: KH: 2,8,Q,K,A (1,2,3,4,1), shp (double-inside)	0.45096
	2-S: KH: 2,8,9,Q,K (1,2,3,4,1), sp (double-inside)	0.45735
	2-S: KH: 2,8,T,Q,K (1,2,3,4,1), sp (inside)	0.45242
46	A	
	1-RF: A: 2,6,7,8,A (1,2,1,3,4), sp	0.46840
	1-RF: A: 6,7,8,T,A (1,2,1,3,4), sp	0.46857
	1-RF: A: 2,3,7,8,A (1,2,1,3,4), 2sp	0.46689
	1-RF: A: 2,6,7,T,A (1,2,1,3,4), 2sp	0.46661
	1-RF: A: 2,3,7,8,A (1,2,1,3,4), 3sp	0.46689
	1-RF: A: 2,5,7,T,A (1,2,1,3,4), 3sp	0.46706
	1-RF: A: 2,6,7,8,A (1,2,3,4,1), sp, fp	0.46172
	1-RF: A: 6,7,8,T,A (1,2,3,4,1), sp, fp	0.46210
	1-RF: A: 6,7,8,T,A (1,2,3,4,4), sp, fp	0.45768

Rank	**Description**	**E[R]**
46	A	
	1-RF: A: 2,3,7,8,A (1,2,3,4,1), 2sp, fp	0.46020
	1-RF: A: 2,6,7,T,A (1,2,3,4,1), 2sp, fp	0.45992
	1-RF: A: 2,6,7,T,A (1,2,3,4,4), 2sp, fp	0.45572
	1-RF: A: 2,3,7,T,A (1,2,3,4,1), 3sp, fp	0.45841
	1-RF: A: 2,5,7,T,A (1,2,3,4,4), 3sp, fp	0.45420
47	2-RF: *KT*: 2,5,8,T,K (1,1,2,3,3)	0.45698
	2-RF: *KT*: 2,5,9,T,K (1,1,2,3,3), sp (double-inside)	0.45205
48	J	
	1-RF: K,Q or J: 2,3,4,6,J (1,2,1,3,4)	0.45812
	1-RF: K,Q or J: 2,3,4,7,J (1,2,1,3,4), sp (double-inside)	0.45633
	1-RF: K,Q or J: 2,3,4,8,J (1,2,1,3,4), sp (inside)	0.45453
	1-RF: K,Q or J: 2,3,4,9,J (1,2,1,3,4), sp (outside)	0.45274
	1-RF: K,Q or J: 2,3,4,T,J (1,2,1,3,4), sp (outside1)	0.45095
	1-RF: K,Q or J: 2,3,7,8,J (1,2,1,3,4), 2sp (double-inside+inside)	0.45336
	1-RF: K,Q or J: 2,3,7,9,J (1,2,1,3,4), 2sp (double-inside+outside)	0.45156
	1-RF: K,Q or J: 2,3,7,T,J (1,2,1,3,4), 2sp (double-inside+outside1)	0.44977
	1-RF: K,Q or J: 2,3,8,9,J (1,2,1,3,4), 2sp (inside+outside)	0.45022
	1-RF: K,Q or J: 2,3,8,T,J (1,2,1,3,4), 2sp (inside+outside1)	0.44842
	1-RF: K,Q or J: 2,3,9,T,J (1,2,1,3,4), 2sp (outside+outside1)	0.44708
	1-RF: K,Q or J: 2,3,4,6,J (1,2,3,4,1), fp	0.45165
	1-RF: K,Q or J: 2,3,4,7,J (1,2,3,4,1), sp (double-inside), fp	0.44985
	1-RF: K,Q or J: 2,3,4,8,J (1,2,3,4,1), sp (inside), fp	0.44806
	1-RF: K,Q or J: 2,3,4,9,J (1,2,3,4,1), sp (outside), fp	0.44626
	1-RF: K,Q or J: 2,3,4,T,J (1,2,3,4,1), sp (outside1), fp	0.44447
	1-RF: K,Q or J: 2,3,7,8,J (1,2,3,4,1), 2sp (double-inside+inside), fp	0.44688
	1-RF: K,Q or J: 2,3,7,9,J (1,2,3,4,1), 2sp (double-inside+outside), fp	0.44509
	1-RF: K,Q or J: 2,3,7,T,J (1,2,3,4,1), 2sp (double-inside+outside1), fp	0.44329
	1-RF: K,Q or J: 2,3,8,9,J (1,2,3,4,1), 2sp (inside+outside), fp	0.44374
	1-RF: K,Q or J: 2,3,8,T,J (1,2,3,4,1), 2sp (inside+outside1), fp	0.44195
	1-RF: K,Q or J: 2,3,9,T,J (1,2,3,4,1), 2sp (outside+outside1), fp	0.44060
	Q	
	1-RF: K,Q or J: 2,5,6,7,Q (1,2,1,3,4)	0.45107
	1-RF: K,Q or J: 2,5,6,8,Q (1,2,1,3,4), sp (double-inside)	0.44928
	1-RF: K,Q or J: 2,5,6,9,Q (1,2,1,3,4), sp (inside)	0.44748
	1-RF: K,Q or J: 2,5,6,T,Q (1,2,1,3,4), sp (outside)	0.44569
	1-RF: K,Q or J: 2,5,8,9,Q (1,2,1,3,4), 2sp (double-inside+inside)	0.44614
	1-RF: K,Q or J: 2,5,8,T,Q (1,2,1,3,4), 2sp (double-inside+inside)	0.44434
	1-RF: K,Q or J: 2,5,9,T,Q (1,2,1,3,4), 2sp (inside+outside)	0.44300
	1-RF: K,Q or J: 2,5,6,7,Q (1,2,3,4,1), fp	0.44459
	1-RF: K,Q or J: 2,5,6,8,Q (1,2,3,4,1), sp (double-inside), fp	0.44280
	1-RF: K,Q or J: 2,5,6,9,Q (1,2,3,4,1), sp (inside), fp	0.44101
	1-RF: K,Q or J: 2,5,6,T,Q (1,2,3,4,1), sp (outside), fp	0.43921
	1-RF: K,Q or J: 2,5,8,9,Q (1,2,3,4,1), 2sp (double-inside+inside), fp	0.43966
	1-RF: K,Q or J: 2,5,8,T,Q (1,2,3,4,1), 2sp (double-inside+inside), fp	0.43787
	1-RF: K,Q or J: 2,5,9,T,Q (1,2,3,4,1), 2sp (inside+outside), fp	0.43652

Rank	**Description**	**E[R]**
48	K	
	1-RF: K,Q or J: 2,5,7,8,K (1,2,1,3,4)	0.44368
	1-RF: K,Q or J: 2,5,7,9,K (1,2,1,3,4), sp (double-inside)	0.44189
	1-RF: K,Q or J: 2,5,7,T,K (1,2,1,3,4), sp (inside)	0.44009
	1-RF: K,Q or J: 2,5,9,T,K (1,2,1,3,4), 2sp (double-inside+inside)	0.43875
	1-RF: K,Q or J: 2,5,7,8,K (1,2,3,4,1), fp	0.43720
	1-RF: K,Q or J: 2,5,7,9,K (1,2,3,4,1), sp (double-inside), fp	0.43541
	1-RF: K,Q or J: 2,5,7,T,K (1,2,3,4,1), sp (inside), fp	0.43362
	1-RF: K,Q or J: 2,5,9,T,K (1,2,3,4,1), 2sp (double-inside+inside), fp	0.43227
49**	4-S: 2,6,8,9,T (1,2,3,4,1) 0H	0.42553
50**	3-F: 2,3,6,8,T (1,2,3,1,1)	0.36355

Appendix C

Joker Wild

Rank	Description	E[R]
Without a joker		
1**	5-RF: T,J,Q,K,A (1,1,1,1,1)	800.00000
2**	5-SF: 2,3,4,5,6 (1,1,1,1,1)	50.00000
3**	5-4K: 9,9,9,9,K (1,2,3,4,1)	23.75000
4	4-RF: *,J,Q,K,A (1,1,1,1,1)	19.79167
	4-RF: *,J,Q,K,A (1,2,2,2,2)	19.89583 (2-9,J,Q)
		19.83333 (T), 19.87500 (K,A)
	4-RF: *,T,Q,K,A (1,1,1,1,1)	19.79167
	4-RF: *,T,Q,K,A (1,2,2,2,2)	19.89583 (2-T,Q)
		19.83333 (J), 19.87500 (K,A)
	4-RF: *,T,J,K,A (1,1,1,1,1)	19.79167
	4-RF: *,T,J,K,A (1,2,2,2,2)	19.89583 (2-T,J)
		19.83333 (Q), 19.87500 (K,A)
	4-RF: *,T,J,Q,A (1,1,1,1,1)	19.72917
	4-RF: *,T,J,Q,A (1,2,2,2,2)	19.83333 (2-Q)
		19.77083 (K), 19.81250 (A)
	4-RF: *,T,J,Q,K (1,1,1,1,1)	20.85417
	4-RF: *,T,J,Q,K (1,2,2,2,2)	20.89583 (2-Q,A), 20.9375 (K)
5**	5-FH: 2,2,T,T,T, (1,2,1,2,3)	7.00000
6**	5-F: 2,4,5,7,T (1,1,1,1,1)	5.00000
7**	4-SF (outside): 2,3,4,5,J (1,1,1,1,2), 0H	4.22917
8**	3-3K: K,K,K,2,4 (1,2,3,1,2)	3.93617
9	4-SF (inside): 2,3,4,6,8 (1,1,1,1,2), 0H	3.10417
	4-SF (inside): 9,T,Q,K,A (1,1,1,1,2), 1H	3.16667
	4-SF (inside): 9,T,Q,K,K (1,1,1,1,2), 1H, 2-HP	3.14583
10	5-S: 2,3,4,5,6 (1,2,3,4,5)	3.00000
11**	4-2P: 2,2,3,3,4 (1,2,1,2,1)	1.62500

Rank	Description	E[R]
12	*KQJ*	
	3-RF: *KQJ*: 3,8,J,Q,K (1,1,2,2,2)	1.47961
	3-RF: *KQJ*: 3,J,Q,K,A (1,2,2,2.3), shp (inside)	1.46454
	3-RF: *KQJ*: 3,9,J,Q,K (1,1,2,2,2), sp (inside)	1.46720
	3-RF: *KQJ*: 3,T,J,Q,K (1,1,2,2,2), sp (outside)	1.45656
	3-RF: *KQJ*: 3,8,J,Q,K (1,2,2,2,2), fp	1.43617
	3-RF: *KQJ*: 8,8,J,Q,K (1,2,3,3,3), 2-LP (2-8)	1.43617
	3-RF: *KQJ*: 3,J,J,Q,K (1,2,3,3,3), 2-LP (J,Q)	1.47252
	3-RF: *KQJ*: 9,J,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.45213
	3-RF: *KQJ*: 3,J,Q,K,A (1,1,1,1,2), shp (inside), fp	1.42110
	3-RF: *KQJ*: J,J,Q,K,A (1,2,2,2,3), shp (inside), 2-LP (J,Q)	1.45745
	3-RF: *KQJ*: 3,9,J,Q,K (1,2,1,1,1), sp (inside), fp	1.42376
	3-RF: *KQJ*: 3,T,J,Q,K (1,2,1,1,1), sp (outside), fp	1.41312
	3-RF: *KQJ*: 9,9,J,Q,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.45479
	3-RF: *KQJ*: 9,J,J,Q,K (1,2,3,3,3), sp (inside), 2-LP (J,Q)	1.46011
	3-RF: *KQJ*: T,T,J,Q,K (1,2,3,3,3), sp (outside), 2-LP (T)	1.43351
	3-RF: *KQJ*: T,J,J,Q,K (1,2,3,3,3), sp (outside), 2-LP (J,Q)	1.44947
	3-RF: *KQJ*: 8,8,J,Q,K (1,2,2,2,2), fp, 2-LP (2-8)	1.43617
	3-RF: *KQJ*: 3,J,J,Q,K (1,2,1,1,1), fp, 2-LP (J,Q)	1.42908
	QJT	
	3-RF: *QJT*: 3,7,T,J,Q (1,1,2,2,2)	1.49468
	3-RF: *QJT*: 3,T,J,Q,A (1,2,2,2,1), shp (inside)	1.47872
	3-RF: *QJT*: 3,T,J,Q,K (1,2,2,2,1), shp (outside)	1.46809
	3-RF: *QJT*: 3,8,T,J,Q (1,1,2,2,2), sp (inside)	1.48138
	3-RF: *QJT*: 3,9,T,J,Q (1,1,2,2,2), sp (outside)	1.47074
	3-RF: *QJT*: 3,7,T,J,Q (1,2,2,2,2), fp	1.45035
	3-RF: *QJT*: 7,7,T,J,Q (1,2,3,3,3), 2-LP (2-7)	1.49468
	3-RF: *QJT*: 7,T,T,J,Q (1,2,3,3,3), 2-LP (T-Q)	1.48404
	3-RF: *QJT*: 8,T,J,Q,A (1,2,2,2,3), shp (inside), sp (inside)	1.46543
	3-RF: *QJT*: 8,T,J,Q,K (1,2,2,2,3), shp (outside), sp (inside)	1.45479
	3-RF: *QJT*: 9,T,J,Q,A (1,2,2,2,3), shp (inside), sp (outside)	1.45479
	3-RF: *QJT*: 3,T,J,Q,A (1,1,1,1,2), shp (inside), fp	1.43440
	3-RF: *QJT*: 3,T,J,Q,K (1,1,1,1,2), shp (outside), fp	1.42376
	3-RF: *QJT*: T,T,J,Q,A (1,2,2,2,3), shp (inside), 2-LP (T-Q)	1.46809
	3-RF: *QJT*: T,T,J,Q,K (1,2,2,2,3), shp (outside), 2-LP (T-Q)	1.45745
	3-RF: *QJT*: 3,8,T,J,Q (1,2,1,1,1), sp (inside), fp	1.43706
	3-RF: *QJT*: 3,9,T,J,Q (1,2,1,1,1), sp (outside), fp	1.42642
	3-RF: *QJT*: 8,8,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (8)	1.46809
	3-RF: *QJT*: 9,9,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (9)	1.44681
	3-RF: *QJT*: 8,T,T,J,Q (1,2,3,3,3), sp (inside), 2-LP (T-Q)	1.47074
	3-RF: *QJT*: 9,T,T,J,Q (1,2,3,3,3), sp (outside), 2-LP (T-Q)	1.46011
	3-RF: *QJT*: 7,7,T,J,Q (1,2,1,1,1), fp, 2-LP (2-7)	1.45035
	3-RF: *QJT*: 7,T,T,J,Q (1,1,2,1,1), fp, 2-LP (T-Q)	1.43972

Rank	Description	E[R]
12	*KJT*	
	3-RF: *KJT*: 3,8,T,J,K (1,1,2,2,2)	1.47961
	3-RF: *KJT*: 3,T,J,K,A (1,2,2,2,3), shp (inside)	1.46454
	3-RF: *KJT*: 8,T,J,Q,K (1,2,2,3,2), sp (outside)	1.45656
	3-RF: *KJT*: 3,9,T,J,K (1,1,2,2,2), sp (inside)	1.46720
	3-RF: *KJT*: 3,8,T,J,K (1,2,2,2,2), fp	1.43617
	3-RF: *KJT*: 8,8,T,J,K (1,2,3,3,3), 2-LP (2-8)	1.47961
	3-RF: *KJT*: 8,T,T,J,K (1,2,3,3,3), 2-LP (T,J)	1.47252
	3-RF: *KJT*: 9,T,J,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.45213
	3-RF: *KJT*: 3,T,J,K,A (1,1,1,1,2), shp (inside), fp	1.42110
	3-RF: *KJT*: 8,T,J,Q,K (1,1,1,2,1), sp (outside), fp	1.41312
	3-RF: *KJT*: 3,9,T,J,K (1,2,1,1,1), sp (inside), fp	1.42376
	3-RF: *KJT*: T,T,J,K,A (1,2,3,3,3), shp (inside), 2-LP (T,J)	1.45475
	3-RF: *KJT*: T,T,J,Q,K (1,2,2,3,2), hp (outside), 2-LP (T,J)	1.44947
	3-RF: *KJT*: 9,9,T,J,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.45479
	3-RF: *KJT*: 9,T,T,J,K (1,2,3,3,3), sp (inside), 2-LP (T,J)	1.46011
	3-RF: *KJT*: 8,8,T,J,K (1,2,1,1,1), fp, 2-LP (2-8)	1.43617
	3-RF: *KJT*: 8,T,T,J,K (1,1,2,1,1), fp, 2-LP (T,J)	1.42908
	KQT	
	3-RF: *KQT*: 3,8,T,Q,K (1,1,2,2,2)	1.47961
	3-RF: *KQT*: 3,T,Q,K,A (1,2,2,2,3), shp (inside)	1.46454
	3-RF: *KQT*: 3,T,J,Q,K (1,2,3,2,2), sp (outisde)	1.45656
	3-RF: *KQT*: 3,9,T,Q,K (1,1,2,2,2), sp (inside)	1.46720
	3-RF: *KQT*: 3,8,T,Q,K (1,2,2,2,2), fp	1.43617
	3-RF: *KQT*: 8,8,T,Q,K (1,2,3,3,3), 2-LP (2-8)	1.47961
	3-RF: *KQT*: 8,T,T,Q,K (1,2,3,3,3), 2-LP (T,Q)	1.47252
	3-RF: *KQT*: 9,T,Q,K,A (1,2,2,2,3), shp (inside), sp (inside)	1.45213
	3-RF: *KQT*: 3,T,Q,K,A (1,1,1,1,2), shp (inside), fp	1.42110
	3-RF: *KQT*: 3,T,J,Q,K (1,1,2,1,1), sp (outisde), fp	1.41312
	3-RF: *KQT*: T,T,Q,K,A (1,2,2,2,3), shp (inside), 2-LP (T,Q)	1.45745
	3-RF: *KQT*: T,T,J,Q,K (1,2,2,2,3), sp (outisde), 2-LP (T,Q)	1.44947
	3-RF: *KQT*: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (9)	1.45479
	3-RF: *KQT*: 9,T,T,Q,K (1,2,3,3,3), sp (inside), 2-LP (T,Q)	1.46011
	3-RF: *KQT*: 3,9,T,Q,K (1,2,1,1,1), sp (inside), fp	1.42376
	3-RF: *KQT*: 9,9,T,Q,K (1,2,3,3,3), sp (inside), 2-LP	1.45479
	3-RF: *KQT*: 8,8,T,Q,K (1,2,1,1,1), fp, 2-LP (2-8)	1.43617
	3-RF: *KQT*: 8,T,T,Q,K (1,1,2,1,1), fp, 2-LP (T,Q)	1.42908
	AKT	
	3-RF: *AKT*: 8,9,T,K,A (1,1,2,2,2)	1.45035
	3-RF: *AKT*: 8,T,Q,K,A (1,2,3,2,2), sp (inside)	1.43794
	3-RF: *AKT*: 8,T,J,K,A (1,2,3,2,2), sp (inside)	1.43794
	3-RF: *AKT*: 8,9,T,K,A (1,2,2,2,2), fp	1.40691
	3-RF: *AKT*: 9,9,T,K,A (1,2,3,3,3), 2-LP (2-9)	1.45035
	3-RF: *AKT*: 9,T,T,K,A (1,2,3,3,3), 2-LP (T)	1.44592
	3-RF: *AKT*: T,Q,Q,K,A (1,2,3,1,1), sp (inside), 2-LP (J,Q)	1.42553
	3-RF: *AKT*: T,T,Q,K,A (1,2,3,1,1), sp (inside), 2-LP (T)	1.44592
	3-RF: *AKT*: T,T,J,K,A (1,2,3,1,1), sp (inside), 2-LP (T)	1.43351
	3-RF: *AKT*: 9,9,T,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.40691
	3-RF: *AKT*: 9,T,T,K,A (1,1,2,1,1), fp, 2-LP (T)	1.40248

Rank	Description	E[R]
12	*AKJ*	
	3-RF: *AKJ*: 8,9,J,K,A (1,1,2,2,2)	1.45035
	3-RF: *AKJ*: 8,J,Q,K,A (1,2,3,3,3), sp (inside)	1.43794
	3-RF: *AKJ*: 3,T,J,K,A (1,2,3,3,3), sp (inside)	1.43794
	3-RF: *AKJ*: 8,9,J,K,A (1,2,2,2,2), fp	1.40691
	3-RF: *AKJ*: 9,9,J,K,A (1,1,2,2,2), 2-LP (2-9)	1.45035
	3-RF: *AKJ*: 9,J,J,K,A (1,1,2,2,2), 2-LP (J)	1.44592
	3-RF: *AKJ*: 8,J,Q,K,A (1,1,3,1,1), sp (inside), fp	1.39450
	3-RF: *AKJ*: 3,T,J,K,A (1,2,1,1,1), sp (inside), fp	1.39450
	3-RF: *AKJ*: J,J,Q,K,A (1,2,3,1,1), sp (inside), 2-LP (J)	1.43351
	3-RF: *AKJ*: T,T,J,K,A (1,2,3,3,3), sp (inside), 2-LP	1.42553
	3-RF: *AKJ*: 9,9,J,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.40691
	3-RF: *AKJ*: 9,J,J,K,A (1,2,1,1,1), fp, 2-LP (J)	1.40248
	AKQ	
	3-RF: *AKQ*: 8,9,Q,K,A (1,1,2,2,2)	1.45035
	3-RF: *AKQ*: 8,J,Q,K,A (1,1,2,2,2), sp (inside)	1.43794
	3-RF: *AKQ*: 8,T,Q,K,A (1,1,2,2,2), sp (inside)	1.43794
	3-RF: *AKQ*: 8,9,Q,K,A (1,2,2,2,2), fp	1.40691
	3-RF: *AKQ*: 9,9,Q,K,A (1,2,2,2,2), 2-LP (2-9)	1.40691
	3-RF: *AKQ*: 9,Q,Q,K,A (1,2,3,3,3), 2-LP (Q)	1.44592
	3-RF: *AKQ*: 8,J,Q,K,A (1,2,1,1,1), sp (inside), fp	1.39450
	3-RF: *AKQ*: 8,T,Q,K,A (1,2,1,1,1), sp (inside), fp	1.39450
	3-RF: *AKQ*: T,T,Q,K,A (1,2,3,3,3), sp (inside), 2-LP (T,J)	1.42553
	3-RF: *AKQ*: T,Q,Q,K,A (1,2,3,3,3), sp (inside), 2-LP (Q)	1.43351
	3-RF: *AKQ*: J,Q,Q,K,A (1,2,3,3,3), sp (inside), 2-LP (Q)	1.43351
	3-RF: *AKQ*: 9,9,Q,K,A (1,2,1,1,1), fp, 2-LP (2-9)	1.40691
	3-RF: *AKQ*: 9,Q,Q,K,A (1,2,1,1,1), fp, 2-LP (Q)	1.40248
13**	2-HP: K,K,5,7,T (1,2,1,2,1)	1.39969
14	*AJT*	
	3-RF: *AJT*: 3,9,T,J,A (1,1,2,2,2)	1.35461
	3-RF: *AJT*: 3,T,J,K,A (1,2,2,3,2), shp (inside)	1.33954
	3-RF: *AJT*: 3,T,J,Q,A (1,2,2,3,2), sp (inside)	1.34220
	3-RF: *AJT*: 3,9,T,J,A (1,2,2,2,2), fp	1.31117
	3-RF: *AJT*: 9,9,T,J,A (1,2,3,3,3), 2-LP (2-9)	1.35461
	3-RF: *AJT*: 3,T,T,J,A (1,1,2,2,2), 2-LP (T,J)	1.34752
	3-RF: *AJT*: T,T,J,K,A (1,2,2,3,2), shp (inside), 2-LP (T,J)	1.33245
	3-RF: *AJT*: T,T,J,Q,A (1,2,2,3,2), sp (inside), 2-LP (T,J)	1.33511
	3-RF: *AJT*: 9,9,T,J,A (1,2,1,1,1), fp, 2-LP (2-9)	1.31117
	3-RF: *AJT*: 3,T,T,J,A (1,2,1,1,1), fp, 2-LP (T,J)	1.30408
	AQT	
	3-RF: *AQT*: 3,9,T,Q,A (1,1,2,2,2)	1.35461
	3-RF: *AQT*: 8,T,Q,K,A (1,2,2,3,2), shp (inside)	1.33954
	3-RF: *AQT*: 8,T,J,Q,A (1,2,3,2,2), sp (inside)	1.34220
	3-RF: *AQT*: 3,9,T,Q,A (1,2,2,2,2), fp	1.31117
	3-RF: *AQT*: 9,9,T,Q,A (1,2,3,3,3), 2-LP (2-9)	1.35461
	3-RF: *AQT*: 9,T,T,Q,A (1,2,3,3,3), 2-LP (T,Q)	1.34752
	3-RF: *AQT*: T,T,Q,K,A (1,2,1,3,1), shp (inside), 2-LP (T,Q)	1.33245
	3-RF: *AQT*: T,T,J,Q,A (1,2,3,1,1), shp (inside), 2-LP (T,Q)	1.33511
	3-RF: *AQT*: 9,9,T,Q,A (1,2,1,1,1), fp, 2-LP (2-9)	1.31117
	3-RF: *AQT*: 9,T,T,Q,A (1,1,2,1,1), fp, 2-LP (T,Q)	1.30408

Rank	Description	E[R]
14	AQJ	
	3-RF: AQJ: 3,8,J,Q,A (1,1,2,2,2)	1.35461
	3-RF: AQJ: 8,J,Q,K,A (1,2,2,3,2), shp (inside)	1.33954
	3-RF: AQJ: 3,T,J,Q,A (1,2,3,3,3), sp (inside)	1.34220
	3-RF: AQJ: 3,8,J,Q,A (1,2,2,2,2), fp	1.31117
	3-RF: AQJ: 9,9,J,Q,A (1,2,3,3,3), 2-LP (2-9)	1.35461
	3-RF: AQJ: 9,J,J,Q,A (1,2,3,3,3), 2-LP (J,Q)	1.34752
	3-RF: AQJ: 3,J,Q,K,A (1,1,1,2,1), shp (inside), fp	1.29610
	3-RF: AQJ: J,J,Q,K,A (1,2,2,2,3), shp (inside), 2-LP (J,Q)	1.33245
	3-RF: AQJ: 3,T,J,Q,A (1,2,1,1,1), sp (inside), fp	1.29876
	3-RF: AQJ: T,T,J,Q,A (1,2,3,3,3), sp (inside), 2-LP (T)	1.32979
	3-RF: AQJ: T,J,J,Q,A (1,2,3,3,3), sp (inside), 2-LP (J,Q)	1.33511
	3-RF: AQJ: 9,9,J,Q,A (1,2,1,1,1), fp, 2-LP (2-9)	1.31117
15	4-F: 2H: 2,4,8,K,A (1,1,2,1,1)	1.16667
	4-F: 1H 4,7,9,T,K (1,2,2,2,2)	1.04170
	4-F: 0H: 3,4,7,8,9 (1,1,1,2,1)	1.04167
16	0H, 3S (outside)	
	3-SF: $s + h = 3$, sp: 4,5,8,9,T (1,2,3,3,3)	0.74645
	3-SF: $s + h = 3$, sp: 4,8,9,T,K (1,2,2,2,3), hp	0.74291
	3-SF: $s + h = 3$, sp: 4,6,8,9,T (1,2,3,3,3), sp (outside)	0.73316
	3-SF: $s + h = 3$, sp: 4,8,9,T,T (1,2,2,2,3), 2-LP	0.73582
	3-SF: $s + h = 3$, sp: 8,8,9,T,K (1,2,2,2,3), hp, 2-LP	0.73227
	3-SF: $s + h = 3$, sp: 6,7,8,K,A (1,1,1,2,3), 2hp	0.73936
17**	2-LP: 2,4,8,T,T (1,2,3,4,1)	0.73183
18	3-SF: $s + h = 3$: 4,9,T,J,K (1,2,2,2,3), shp (inside)	0.72784
	3-SF: $s + h = 3$: 9,T,J,K,A (1,1,1,2,3), shp (inside), hp	0.72429
	3-SF: $s + h = 3$: 8,9,T,J,K (1,2,2,2,3), shp (inside), sp (inside)	0.70390
	3-SF: $s + h = 3$: 6,8,9,T,K (1,2,2,2,3), hp, sp (outside)	0.72961
19**	4-S: KQJT: 3,T,J,Q,K (1,2,3,4,1)	0.62500
20	0H, 2S (inside)	
	3-SF: $s + h = 2$: 4,5,7,9,T (1,2,3,3,3)	0.61879
	3-SF: $s + h = 2$: 4,9,J,Q,K (1,2,2,2,3), shp (inside)	0.60018
	3-SF: $s + h = 2$: 4,9,J,Q,A (1,2,2,2,3), hp	0.61259
	3-SF: $s + h = 2$: 4,9,T,J,Q (1,2,3,2,2), sp (outside)	0.59220
	3-SF: $s + h = 2$: 4,8,9,J,Q (1,2,3,3,3), sp (inside)	0.60284
	3-SF: $s + h = 2$, : 6,8,9,K,A (1,1,1,2,3), 2hp	0.61170
	3-SF: $s + h = 2$: 9,J,Q,K,A (1,1,1,2,3), shp (inside), hp	0.59663
	3-SF: $s + h = 2$: 7,8,9,J,A (1,2,2,2,3), hp, sp (inside)	0.60195
	3-SF: $s + h = 2$: 8,9,T,J,A (1,1,2,1,3), hp, sp (outside)	0.59131
	1H, 1S (inside)	
	3-SF: $s + h = 2$: 4,5,9,J,K (1,2,3,3,3)	0.60106
	3-SF: $s + h = 2$: 4,9,J,K,A (1,2,2,2,3), hp	0.59840
	3-SF: $s + h = 2$: 4,9,J,Q,K (1,2,2,3,2), sp (inside)	0.58865
	3-SF: $s + h = 2$: 9,J,Q,K,A (1,1,2,1,3), hp (inside), sp (inside)	0.58599

Rank	Description	E[R]
21	2-RF: AK: 3,8,9,K,A (1,2,3,4,4)	0.57661
	2-RF: AK: 3,8,J,K,A (1,2,3,4,4), sp (double-inside)	0.57291
	2-RF: AK: 8,J,Q,K,A (1,2,3,1,1), 2sp (both double-inside)	0.57002
	2-RF: AK: 3,8,9,K,A (1,2,3,1,1), fp	0.56129
	2-RF: AK: 3,8,J,K,A (1,2,3,1,1), sp (double-inside), fp	0.55759
	2-RF: AK: 8,J,Q,K,A (1,2,3,1,1), 2sp (both double-inside), fp	0.55469
22**	4-S: 5432–QJT9: 2,9,T,J,Q (1,2,3,4,1)	0.56250
23**	3-SF: $s + h = 1$: 2,3,5,7,9 (1,2,3,3,3)	0.49113
	3-SF: $s + h = 1$: 3,4,7,9,A (1,1,1,2,3), hp	0.48759
	3-SF: $s + h = 1$: 3,4,7,K,A (1,1,1,2,3), 2hp	0.48404
24**	2-S: AK if one of (KQ,KJ,KT) fp and 9sp: 7,9,Q,K,A (1,2,1,1,3)	0.45057
25	KQ	
	2-RF: KQ: 2,4,6,Q,K (1,2,3,4,4)	0.47930
	2-RF: KQ: 2,8,Q,K,A (1,2,3,3,4), shp (double-inside)	0.46936
	2-RF: KQ: 2,J,Q,K,A (1,2,3,3,4), shp (double-inside), sp (inside)	0.46323
	2-RF: KQ: 2,9,J,Q,K (1,2,3,4,4), sp (double-inside), sp (inside)	0.46947
	2-RF: KQ: 2,8,J,Q,K (1,2,3,4,4), sp (inside)	0.46236
	2-RF: KQ: 2,4,6,Q,K (1,2,3,1,1), fp	0.46398
	2-RF: KQ: 2,8,Q,K,A (1,2,1,1,3), shp (double-inside), fp	0.45404
	2-RF: KQ: 2,8,J,Q,K (1,2,3,1,1), sp (insie), fp	0.45704
	2-RF: KQ: 7,J,Q,K,A (1,2,3,3,4), shp (double-inside), sp (inside), fp	0.44791
	2-RF: KQ: 9,J,Q,K,A (1,2,3,3,4), shp (double-inside), 2sp (double-inside+inside)	0.46034
	KJ	
	2-RF: KJ: 2,4,6,J,K (1,2,3,4,4)	0.47930
	2-RF: KJ: 2,8,J,K,A (1,2,3,3,4), shp (double-inside)	0.46936
	2-RF: KJ: 2,J,Q,K,A (1,2,3,2,4), shp (double-inside), sp (inside)	0.46323
	2-RF: KJ: 2,9,J,Q,K (1,2,3,4,3), sp (double-inside), sp (inside)	0.46947
	2-RF: KJ: 2,8,J,Q,K (1,2,3,4,3), sp (inside)	0.46236
	2-RF: KJ: 2,4,6,J,K (1,2,3,1,1), fp	0.46398
	2-RF: KJ: 2,8,J,K,A (1,2,1,1,3), shp (double-inside), fp	0.45404
	2-RF: KJ: 2,8,J,Q,K (1,2,1,2,1), sp (inside), fp	0.45704
	2-RF: KJ: 7,J,Q,K,A (1,2,3,2,4), shp (double-inside), sp (inside), fp	0.44791
	2-RF: KJ: 9,J,Q,K,A (1,2,3,2,4), shp (double-inside), 2sp (double-inside+inside)	0.46034
	KT	
	2-RF: KT: 2,4,6,T,K (1,2,3,4,4)	0.47560
	2-RF: KT: 2,4,T,K,A (1,2,3,3,4), shp (double-inside)	0.46566
	2-RF: KT: 2,4,T,Q,K (1,2,3,3,4), sp (inside)	0.46947
	2-RF: KT: 2,4,9,T,K (1,2,3,4,4), sp (double-inside)	0.47560
	2-RF: KT: 2,4,6,T,K (1,2,3,1,1), fp	0.46398
	2-RF: KT: 2,T,Q,K,A (1,2,3,2,4), shp (double-inside), sp (inside)	0.46323
	2-RF: KT: 9,T,Q,K,A (1,2,3,2,4), shp (double-inside), 2sp (double-inside+inside)	0.46304
	2-RF: KT: 2,6,T,K,A (1,2,3,3,4), shp (inside), fp	0.45404
	2-RF: KT: 2,9,T,Q,K (1,2,3,3,4), 2sp (double-inside+inside)	0.46566
	2-RF: KT: 2,4,T,Q,K (1,2,1,3,1), sp (inside), fp	0.45704
	2-RF: KT: 9,T,Q,K,A (1,2,3,2,4), shp (double-inside), 2sp (double-inside+inside)	0.46034
	2-RF: KT: 2,T,Q,K,A (1,1,2,1,3), shp (double-inside), sp (inside), fp	0.44791
	2-RF: KT: 2,9,T,Q,K (1,2,1,3,1), 2sp (double-inside+inside), fp	0.45415
26**	2-S: AK: one of (AKQJ,AKQT,AKJT) if one of (AQ,AJ,AT), 8,J,Q,K,A (1,2,3,4,2)	0.44768

Rank	Description	E[R]
27**	1-RF: A: AKQJx, AKQTx, AKJTx if x is 6-9: 9,J,Q,K,A (1,2,3,4,1)	0.44801
28	2-S: AK: 2,4,8,K,A (1,2,3,4,1)	0.45427
	2-S: AK: 2,4,Q,K,A (1,2,3,4,1), sp (double-inside)	0.45057
	2-S: AK: 2,J,Q,K,A (1,2,3,4,1), 2sp (both double-inside)	0.44768
	2-S: AK: 2,4,8,K,A (1,2,1,3,1), fp	0.45427
	2-S: AK: 2,7,Q,K,A (1,1,3,4,1), sp (double-inside), fp	0.45057
	2-S: AK: 2,J,Q,K,A (1,1,2,3,1), 2sp (both double-inside)	0.44768
29	1-RF: A if one of (AQ,AJ,AT) is fp: 2,4,7,Q,A (1,2,3,1,1)	0.44229
	1-RF: A if one of (AQ,AJ,AT) is fp: 2,4,J,Q,A (1,2,3,1,1), sp (double-inside)	0.44113
	1-RF: A if one of (AQ,AJ,AT) is fp: 2,T,J,Q,A (1,2,3,1,1), 2sp (double-inside)	0.44139
30	2-RF: AQ,AJ,AT: 2,4,7,Q,A (1,2,3,4,4)	0.45710
	2-RF: AQ,AJ,AT: 2,4,J,Q,A (1,2,3,4,4), sp (double-inside)	0.45340
	2-RF: AQ,AJ,AT: 2,T,J,Q,A (1,2,3,4,4), 2sp (double-inside)	0.45051
31	K	
	1-RF: A or K: 2,5,7,8,K (1,2,1,3,4)	0.46128
	1-RF: A or K: 2,5,7,9,K (1,2,1,3,4), sp (double-inside)	0.45980
	1-RF: A or K: 2,5,7,T,K (1,2,1,3,4), sp (inside)	0.45848
	1-RF: A or K: 2,5,9,T,K (1,2,1,3,4), 2sp (double-inside+inside)	0.45734
	1-RF: A or K: 2,5,7,8,K (1,2,3,4,1), fp	0.45591
	1-RF: A or K: 2,5,7,9,K (1,2,3,4,1), sp (double-inside), fp	0.45443
	1-RF: A or K: 2,5,7,T,K (1,2,3,4,1), sp (inside), fp	0.45311
	1-RF: A or K: 2,5,9,T,K (1,2,3,4,1), 2sp (double-inside+inside), fp	0.45196
	1-RF: A or K: 2,5,7,8,K (1,1,2,3,1), fp	0.45591
	1-RF: A or K: 2,5,7,9,K (1,1,2,3,1), sp (double-inside), fp	0.45190
	1-RF: A or K: 2,5,7,9,K (1,2,3,1,1), sp (double-inside), fp	0.45190
	1-RF: A or K: 2,5,7,T,K (1,1,2,3,1), sp (inside), fp	0.44911
	1-RF: A or K: 2,5,9,T,K (1,1,2,3,1), 2sp (double-inside+inside), fp	0.44795
	1-RF: A or K: 2,5,9,T,K (1,2,1.3,1), 2sp (double-inside+inside), fp	0.44708
	A	
	1-RF: A or K: 2,6,7,8,A (1,2,1,3,4), sp	0.46068
	1-RF: A or K: 6,7,8,T,A (1,2,1,3,4), sp	0.46068
	1-RF: A or K: 2,3,7,8,A (1,2,1,3,4), 2sp	0.45953
	1-RF: A or K: 2,6,7,T,A (1,2,1,3,4), 2sp	0.45920
	1-RF: A or K: 2,3,7,8,A (1,2,1,3,4), 3sp	0.45953
	1-RF: A or K: 2,5,7,T,A (1,2,1,3,4), 3sp	0.45805
	1-RF: A or K: 2,3,4,T,A (1,2,1,3,4), 4sp	0.45715
	1-RF: A or K: 2,6,7,8,A (1,2,3,4,1), sp, fp	0.45443
	1-RF: A or K: 6,7,8,T,A (1,2,3,4,1), sp, fp	0.45531
	1-RF: A or K: 6,7,8,T,A (1,2,3,4,4), sp, fp	0.45710
	1-RF: A or K: 2,3,7,8,A (1,2,3,4,1), 2sp, fp	0.45328
	1-RF: A or K: 2,6,7,T,A (1,2,3,4,1), 2sp, fp	0.45180
	1-RF: A or K: 2,5,7,T,A (1,2,3,4,1), 3sp, fp	0.45180
	1-RF: A or K: 2,3,4,T,A (1,2,3,4,1), 4sp, fp	0.45090

Rank	**Description**	**E[R]**
32	2-RF: JT: 2,5,6,T,J (1,2,3,4,4)	0.38270
	2-RF: JT: 2,5,T,J,Q (1,2,3,3,4), hp (outside)	0.37205
	2-RF: JT: 2,5,7,T,J (1,2,3,4,4), sp (double-inside)	0.37853
	2-RF: JT: 2,5,8,T,J (1,2,3,4,4), sp (inside)	0.37853
	2-RF: JT: 2,5,9,T,J (1,2,3,4,4), sp (outside)	0.37182
	2-RF: JT: 2,5,6,T,J (1,2,3,1,1), fp	0.36691
	2-RF: JT: 2,7,T,J,Q (1,2,3,3,4), hp (outiside), sp (double-inside)	0.36789
	2-RF: JT: 2,8,T,J,Q (1,2,3,3,4), hp (outiside), sp (double-inside)	0.36589
	2-RF: JT: 2,5,T,J,Q (1,2,1,1,3), hp (outside), fp	0.35627
	2-RF: JT: 2,7,8,T,J (1,2,3,4,4), 2sp (double-inside+inside)	0.37176
	2-RF: JT: 2,7,9,T,J (1,2,3,4,4), 2sp (double-inside+outside)	0.36852
	2-RF: JT: 2,7,8,T,J (1,2,3,1,1), 2sp (double-inside+inside), fp	0.35598
	2-RF: JT: 2,7,9,T,J (1,2,3,1,1), 2sp (outside+inside), fp	0.35274
33	Discard everything if QT fp with J or QT fp with 2sp	
	2,5,T,J,Q (1,2,3,2,2)	0.33296
	2,8,T,J,Q (1,2,3,2,2)	0.33345
	2,8,9,T,Q (1,2,3,1,1)	0.33261
34	2-RF: QT: 2,4,6,T,Q (1,2,3,4,4)	0.35875
	2-RF: QT: 2,4,T,J,Q (1,2,3,4,3), sp (outside)	0.34812
	2-RF: QT: 2,4,7,T,Q (1,2,3,4,4), sp (double-inside)	0.35875
	2-RF: QT: 2,4,8,T,Q (1,2,3,4,4), sp (inside)	0.35459
	2-RF: QT: 2,4,9,T,Q (1,2,3,4,4), sp (outside)	0.35135
	2-RF: QT: 2,8,T,J,Q (1,2,3,4,3), 2sp (outside+inside)	0.34482
	2-RF: QT: 2,8,9,T,Q (1,2,3,4,4), 2sp (inside+outside)	0.34806
	2-RF: QT: 2,4,6,T,Q (1,2,3,1,1), fp	0.34297
	2-RF: QT: 2,4,8,T,Q (1,2,3,1,1), sp (inside), fp	0.33881
	2-RF: QT: 2,4,9,T,Q (1,2,3,1,1), sp (outside), fp	0.33557

Rank	Description	E[R]
With a joker		
1**	5-K: 1,2,2,2,2 (0,1,2,3,4)	200.00000
2**	5-RF: 1,T,J,Q,K (0,1,1,1,1)	100.00000
3**	5-SF: 1,2,3,4,5 (0,1,1,1,1)	50.00000
4**	4-K: 1,2,2,2,9 (0,1,2,3,1)	23.75000
5	*QJT*	
	4-RF: *QJT*: 1,4,T,J,Q (0,1,2,2,2)	8.00000
	4-RF: *QJT*: 1,T,J,Q,A (0,1,1,1,2) shp or sp	7.93750
	4-RF: *QJT*: 1,T,T,J,Q (0,1,2,2,2), 2-LP	7.95833
	4-RF: *QJT*: 1,4,T,J,Q (0,1,1,1,1), fp	7.89583
	KQJ	
	4-RF: *KQJ*: 1,4,J,Q,K (0,1,2,2,2)	7.29167
	4-RF: *KQJ*: 1,J,Q,K,A (0,1,1,1,2) shp or sp	7.25000
	4-RF: *KQJ*: 1,J,J,Q,K (0,1,2,2,2), 2-LP or 2-HP	7.27083
	4-RF: *KQJ*: 1,4,J,Q,K (0,1,1,1,1), fp	7.20833
	KJT	
	4-RF: *KJT*: 1,4,T,J,K (0,1,2,2,2)	7.29167
	4-RF: *KJT*: 1,T,J,K,A (0,1,1,1,2) shp or sp	7.25000
	4-RF: *KJT*: 1,4,T,J,K (0,1,2,2,2), 2-LP or 2-HP	7.27083
	4-RF: *KJT*: 1,4,T,J,K (0,1,1,1,1), fp	7.20833
	KQT	
	4-RF: *KQT*: 1,4,T,Q,K (0,1,2,2,2)	7.29167
	4-RF: *KQT*: 1,T,Q,K,A (0,1,1,1,2) shp or sp	7.25000
	4-RF: *KQT*: 1,T,T,Q,K (0,1,2,2,2), 2-LP or 2-HP	7.27083
	4-RF: *KQT*: 1,4,T,Q,K (0,1,1,1,1), fp	7.20833
6**	5-FH: 1,T,T,K,K (0,1,2,1,2)	7.00000
7	*AKT*	
	4-RF: *AKT*: 1,4,T,K,A (0,1,2,2,2)	6.22917
	4-RF: *AKT*: 1,T,Q,K,A (0,1,2,1,1), sp	6.18750
	4-RF: *AKT*: 1,T,T,K,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AKT*: 1,4,T,K,A (0,1,1,1,1), fp	6.14583
	AQT	
	4-RF: *AQT*: 1,4,T,Q,A (0,1,2,2,2)	6.22917
	4-RF: *AQT*: 1,T,Q,K,A (0,1,1,2,1), sp	6.18750
	4-RF: *AQT*: 1,T,T,Q,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AQT*: 1,4,T,Q,A (0,1,1,1,1), fp	6.14583
	AJT	
	4-RF: *AJT*: 1,4,T,J,A (0,1,2,2,2)	6.22917
	4-RF: *AJT*: 1,T,J,K,A (0,1,1,2,1), sp	6.18750
	4-RF: *AJT*: 1,T,T,J,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AJT*: 1,4,T,J,A (0,1,1,1,1), fp	6.14583
	AKQ	
	4-RF: *AKQ*: 1,4,Q,K,A (0,1,2,2,2)	6.22917
	4-RF: *AKQ*: 1,J,Q,K,A (0,1,1,2,1), sp	6.18750
	4-RF: *AKQ*: 1,Q,Q,K,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AKQ*: 1,4,Q,K,A (0,1,1,1,1), fp	6.14583

Rank	**Description**	**E[R]**
7	*AKJ*	
	4-RF: *AKJ*: 1,4,J,K,A (0,1,2,2,2)	6.22917
	4-RF: *AKJ*: 1,J,Q,K,A (0,1,1,2,1), sp	6.18750
	4-RF: *AKJ*: 1,J,J,K,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AKJ*: 1,4,Q,K,A (0,1,1,1,1), fp	6.14583
	AKT	
	4-RF: *AKT*: 1,4,T,K,A (0,1,2,2,2)	6.22917
	4-RF: *AKT*: 1,T,J,K,A (0,1,2,1,1), sp	6.18750
	4-RF: *AKT*: 1,T,T,K,A (0,1,2,2,2), 2-LP or 2-HP	6.20833
	4-RF: *AKT*: 1,4,T,J,A (0,1,1,1,1), fp	6.14583
8	4-SF (outside, $s = 3$): 1,3,4,5,T (0,1,1,1,2)	5.97917
	4-SF (outside, $s = 3$): 1,3,4,5,K (0,1,1,1,2), hp	5.95833
	4-SF (outside, $s = 3$): 1,3,4,5,6 (0,1,1,1,2), sp (outside)	5.91667
	4-SF (outside, $s = 3$): 1,3,4,5,7 (0,1,1,1,2), sp (intside)	5.91667
	4-SF (outside, $s = 3$): 1,3,4,5,5 (0,1,1,1,2), 2-LP	5.93750
	4-SF (outside, $s = 3$): 1,3,4,5,T (0,1,1,1,1), fp	5.87500
9**	5-F: 1,3,5,9,T (0,1,1,1,1)	5.00000
10	4-SF (inside, $s = 1,2$): 1,3,4,6,T (0,1,1,1,2)	4.91667
	4-SF (inside, $s = 1,2$): 1,3,4,6,K (0,1,1,1,2), hp	4.89583
	4-SF (inside, $s = 1,2$): 1,3,4,5,6 (0,1,1,2,1), sp (outside)	4.85417
	4-SF (inside, $s = 1,2$): 1,3,4,6,7 (0,1,1,2,1), sp (inside)	4.85417
	4-SF (inside, $s = 1,2$): 1,3,4,6,6 (0,1,1,1,2), 2-LP	4.87500
11**	3-3K: 1,3,3,5,T (0,1,2,1,2)	3.93617
12	5-S: 1,2,3,4,5, (0,1,2,3,4)	3.00000
13	*KQ*	
	3-RF: *KQ*: 1,2,5,Q,K (0,1,2,3,3)	2.08599
	3-RF: *KQ*: 1,5,Q,K,A (0,1,2,2,3), shp	2.07181
	3-RF: *KQ*: 1,2,J,Q,K (0,1,2,3,3), sp	2.06472
	3-RF: *KQ*: 1,2,5,Q,K (0,1,2,1,1), fp	2.05053
	3-RF: *KQ*: 1,5,Q,K,A (0,1,1,1,2), shp, fp	2.03635
	3-RF: *KQ*: 1,2,J,Q,K (0,1,2,1,1), sp, fp	2.02926
	KJ	
	3-RF: *KJ*: 1,2,5,J,K (0,1,2,3,3)	2.08599
	3-RF: *KJ*: 1,5,J,K,A (0,1,2,2,3), shp	2.07181
	3-RF: *KJ*: 1,2,J,Q,K (0,1,2,3,2), sp	2.06472
	3-RF: *KJ*: 1,2,5,J,K (0,1,2,1,1), fp	2.05053
	3-RF: *KJ*: 1,5,J,K,A (0,1,1,1,2), shp, fp	2.03635
	3-RF: *KJ*: 1,2,J,Q,K (0,1,1,2,1), sp, fp	2.02926
	KT	
	3-RF: *KT*: 1,2,5,T,K (0,1,2,3,3)	2.08599
	3-RF: *KT*: 1,5,T,K,A (0,1,2,2,3), shp	2.07181
	3-RF: *KT*: 1,2,T,Q,K (0,1,2,3,2), sp	2.06472
	3-RF: *KT*: 1,2,5,T,K (0,1,2,1,1), fp	2.05053
	3-RF: *KT*: 1,5,T,K,A (0,1,1,1,2), shp, fp	2.03635
	3-RF: *KT*: 1,2,T,Q,K (0,1,1,2,1), sp, fp	2.02926
14**	4-F: *HHx* or *Hxx*: 1,2,6,J,K (0,1,1,2,1)	2.02083

Rank	Description	E[R]
15	*JT*	
	3-RF *JT*: 1,2,3,J,T (0,1,2,3,3)	1.99557
	3-RF: *JT*: 1,2,J,Q,A (0,1,2,2,3), shp (double-inside)	1.95390
	3-RF: *JT*: 1,2,T,J,K (0,1,2,2,3), shp (inside)	1.94681
	3-RF: *JT*: 1,2,T,J,Q (0,1,2,2,3), sp (outside)	1.96011
	3-RF: *JT*: 1,2,9,T,J (0,1,2,3,3), sp (outside)	1.95656
	3-RF: *JT*: 1,2,8,T,J (0,1,2,3,3), sp (inside)	1.96365
	3-RF: *JT*: 1,2,7,T,J (0,1,2,3,3), sp (double-inside)	1.97429
	3-RF: *JT*: 1,2,3,T,Q (0,1,2,1,1), fp	1.95301
	3-RF: *JT*: 1,2,T,J,A (0,1,1,1,2), shp (double-inside), fp	1.91135
	3-RF: *JT*: 1,2,T,J,K (0,1,1,1,2), shp (inside), fp	1.90426
	3-RF: *JT*: 1,2,T,J,Q (0,1,1,1,2), sp (outside), fp	1.91755
	3-RF: *JT*: 1,2,9,T,J (0,1,2,1,1), sp (outside), fp	1.91401
	3-RF: *JT*: 1,2,8,T,J (0,1,2,1,1), sp (inside), fp	1.92110
	3-RF: *JT*: 1,2,7,T,J (0,1,2,1,1), sp (double-inside), fp	1.93174
	AK	
	3-RF: *AK*: 1,2,5,K,A (0,1,2,3,3)	1.95301
	3-RF: *AK*: 1,2,Q,K,A (0,1,2,3,3), sp	1.93883
	AQ	
	3-RF: *AQ*: 1,2,4,Q,A (0,1,2,3,3)	1.95301
	3-RF: *AQ*: 1,2,Q,K,A (0,1,2,3,2), shp or sp	1.93883
	AJ	
	3-RF: *AJ*: 1,2,4,J,A (0,1,2,3,3)	1.95301
	3-RF: *AJ*: 1,2,J,K,A (0,1,2,3,2), shp or sp	1.93883
	AT	
	3-RF: *AT*: 1,2,4,T,A (0,1,2,3,3)	1.95301
	3-RF: *AT*: 1,2,T,K,A (0,1,2,3,2), shp or sp	1.93883
16	3-SF (outside): 1,4,5,T,J (0,1,1,2,3)	1.88918
	3-SF (outside): 1,4,5,T,A (0,1,1,2,3), shp	1.84752
	3-SF (outside): 1,4,5,T,K (0,1,1,2,3), hp	1.86170
	3-SF (outside): 1,2,4,5,J (0,1,2,2,3), sp (inside)	1.86082
	3-SF (outside): 1,3,4,5,J (0,1,2,2,3), sp (outside)	1.85078
	3-SF (outside): 1,2,5,6,J (0,1,2,2,3), sp (double-inside)	1.89450
	3-SF (outside): 1,4,5,T,J (0,1,1,1,2), fp	1.84663
	3-SF (outside): 1,4,5,K,A (0,1,1,2,3), shp, hp	1.81915
	3-SF (outside): 1,4,5,7,A (0,1,1,2,3), shp, sp (inside)	1.81560
	3-SF (outside): 1,4,5,6,A (0,1,1,2,3), shp, sp (outside)	1.80496
	3-SF (outside): 1,4,5,8,A (0,1,1,2,3), shp, sp (double-inside)	1.82624
	3-SF (outside): 1,4,5,T,A (0,1,1,1,2), shp, fp	1.80496
	3-SF (outside): 1,6,7,K,A (0,1,1,2,3), 2hp	1.85993
	3-SF (outside): 1,6,7,9,A (0,1,1,2,3), hp, sp (inside)	1.85638
	3-SF (outside): 1,6,7,8,A (0,1,1,2,3), hp, sp (outside)	1.84574
	3-SF (outside): 1,6,7,T,A (0,1,1,2,3), hp, sp (double-inside)	1.86702
	3-SF (outside): 1,6,7,J,A (0,1,1,1,2), hp, fp	1.84574

Rank	Description	E[R]
17	3-SF with A: 1,2,6,9,A (0,1,2,3,1)	1.82004
	3-SF with A: 1,2,3,9,A (0,1,2,3,1), any sp	1.80585
	3-SF with K: 1,2,6,9,K (0,1,2,3,3)	1.82004
	3-SF with K: 1,2,9,T,K (0,1,2,3,2), any sp	1.80585
18	QJ	
	3-RF: QJ: 1,2,3,J,Q (0,1,2,3,3)	1.83599
	3-RF: QJ: 1,2,J,Q,A (0,1,2,2,3), shp (inside)	1.79433
	3-RF: QJ: 1,2,J,Q,K (0,1,2,3,3), shp (outside)	1.78723
	3-RF: QJ: 1,2,T,J,Q (0,1,2,3,3), sp (outside)	1.80053
	3-RF: QJ: 1,2,9,J,Q (0,1,2,3,3), sp	1.80762
	3-RF: QJ: 1,2,3,J,Q (0,1,2,1,1), fp	1.79344
	3-RF: QJ: 1,2,J,Q,A (0,1,1,1,2), shp (inside), fp	1.75177
	3-RF: QJ: 1,2,J,Q,K (0,1,1,1,2), shp (outside), fp	1.74468
	3-RF: QJ: 1,2,T,J,Q (0,1,2,1,1), sp (outside), fp	1.75798
	QT	
	3-RF: QT: 1,2,3,T,Q (0,1,2,3,3)	1.83599
	3-RF: QT: 1,2,T,Q,A (0,1,2,2,3), shp (double-inside)	1.79433
	3-RF: QT: 1,2,T,Q,K (0,1,2,3,3), shp (inside)	1.78723
	3-RF: QT: 1,2,T,J,Q (0,1,2,3,2), sp (outside)	1.80053
	3-RF: QT: 1,2,9,T,Q (0,1,2,3,3), sp (inside)	1.80762
	3-RF: QT: 1,2,8,T,Q (0,1,2,3,3), sp (double-inside)	1.81472
	3-RF: QT: 1,2,3,T,Q (0,1,2,1,1), fp	1.79344
	3-RF: QT: 1,2,T,Q,A (0,1,1,1,2), shp (double-inside), fp	1.75177
	3-RF: QT: 1,2,T,Q,K (0,1,1,1,2), shp (inside), fp	1.74468
	3-RF: QT: 1,2,T,J,Q (0,1,1,2,1), sp (outside), fp	1.75798
	3-RF: QT: 1,2,9,T,Q (0,1,2,1,1), sp (inside), fp	1.76507
	3-RF: QT: 1,2,8,T,Q (0,1,2,1,1), sp (double-inside), fp	1.77216
19	2-HP: K if 4-S with A or A fp: 1,2,5,K,A (0,1,1,2,3)	1.70496
	2-HP: K if 4-S with A or A fp: 1,6,8,K,A (0,1,2,3,1)	1.70496
	2-HP: K if 4-S with A or A fp: 1,2,8,K,A (0,1,2,3,2)	1.70496
	2-HP: K if 4-S with A or A fp: 1,2,9,K,A (0,1,2,3,2) sp	1.69941
20	2-HP: A if AK: 1,6,7,K,A (0,1,2,3,4)	1.71485
	2-HP: A if AK: 1,2,7,K,A (0,1,2,3,4), shp, sp	1.70930
	2-HP: A if AK: 1,6,J,K,A (0,1,1,2,3), sp	1.71022
	2-HP: A if AK: 1,3,J,K,A (0,1,1,2,3), 2sp	1.70467
21**	3-SF (inside: $46–8T$): if no sp, no fp: 1,4,6,9,A (0,1,1,2,2), hp	1.72872
22	2-HP: A: 1,6,7,8,A (0,1,2,3,4)	1.72040
	2-HP: A: 1,2,7,8,A (0,1,2,3,4), sp	1.71485
	2-HP: A: 1,2,7,K,A (0,1,2,3,4), shp, sp	1.70930
	2-HP: A: 1,2,3,8,A (0,1,2,3,4), 2sp	1.71022
	2-HP: A: 1,2,T,K,A (0,1,2,3,4), shp, 2sp	1.70467
	2-HP: A: 1,6,8,9,A (0,1,2,3,2), fp	1.70768
	2-HP: A: 1,2,8,9,A (0,1,2,3,2), sp, fp	1.70213
	2-HP: A: 1,2,3,9,A (0,1,1,2,3), 2sp, fp	1.69750
23**	4-S: KQJ, KQT, KJT: 1,2,J,Q,K (0,1,2,3,1)	1.68750

Rank	Description	E[R]
24	2-HP: K: 1,2,4,6,K (0,1,2,3,4)	1.71051
	2-HP: K: 1,2,4,Q,K (0,1,2,3,4), sp (inside or outside)	1.70126
	2-HP: K: 1,2,4,9,K (0,1,2,3,4), sp (double-inside)	1.70496
	2-HP: K: 1,2,9,Q,K (0,1,2,3,4) 2sp (double-inside+inside)	1.69664
	2-HP: K: 1,2,J,Q,K (0,1,2,3,4) 2sp (outside or inside)	1.69339
	2-HP: K: 1,2,4,7,K (0,1,2,3,1), fp	1.69779
	2-HP: K: 1,2,4,Q,K (0,1,2,3,1), sp (inside or outside), fp	1.68854
	2-HP: K: 1,2,4,9,K (0,1,2,3,1), sp (double-inside), fp	1.69224
	2-HP: K: 1,2,9,Q,K (0,1,2,3,1) 2sp (double-inside+inside), fp	1.68392
25	3-SF (inside *46–8T*): 1,4,6,T,J (0,1,1,2,3)	1.75621
	3-SF (inside *46–8T*): 1,4,6,7,T (0,1,1,2,3), sp (inside)	1.72429
	3-SF (inside *46–8T*): 1,4,5,6,T (0,1,2,1,3), sp (outside)	1.71365
	3-SF (inside *46–8T*): 1,4,6,8,T (0,1,1,2,3), sp (double-inside)	1.73493
	3-SF (inside *46–8T*): 1,4,6,T,J (0,1,1,1,2), fp	1.75621
	3-SF (inside *46–8T*): 1,4,6,7,T (0,1,1,2,1), sp (inside), fp	1.68174
	3-SF (inside *46–8T*): 1,4,5,6,T (0,1,2,1,1), sp (outside), fp	1.67110
	3-SF (inside *46–8T*): 1,4,6,8,T (0,1,1,2,1), sp (double-inside), fp	1.69238
	3-SF (inside *35* or *9J*): 1,3,5,T,J (0,1,1,2,3)	1.72961
	3-SF (inside *35* or *9J*): 1,3,5,6,J (0,1,1,2,3), sp (inside)	1.69770
	3-SF (inside *35* or *9J*): 1,3,4,5,J (0,1,2,1,3), sp (outside)	1.69060
	3-SF (inside *35* or *9J*): 1,3,5,7.J (0,1,2,1,3), sp (double-inside)	1.70833
	3-SF (inside *35* or *9J*): 1,3,5,T,J (0,1,1,1,2), fp	1.68706
	3-SF (inside *35* or *9J*): 1,3,5,6,J (0,1,1,2,1), sp (inside), fp	1.65514
	3-SF (inside *35* or *9J*): 1,3,4,5,J (0,1,2,1,1), sp (outside), fp	1.64805
	3-SF (inside *35* or *9J*): 1,3,5,7.J (0,1,2,1,1), sp (double-inside), fp	1.66578
26	3-SF (inside *25–9Q*) if no fp	
	3-SF (inside *36–8J*): 1,4,7,9,J (0,1,1,2,3)	1.59663
	3-SF (inside *36–8J*): 1,4,7,8,J (0,1,1,2,3), sp (double-inside)	1.57535
	3-SF (inside *36–8J*): 1,4,5,7,J (0,1,2,1,3), sp (inside)	1.56472
	3-SF (inside *36–8J*): 1,4,7,10,J (0,1,1,1,2), fp	1.55408
	3-SF (inside *25* or *9Q*): 1,2,5,9,J (0,1,1,2,3)	1.57004
	3-SF (inside *25* or *9Q*): 1,2,5,6,J (0,1,1,2,3), sp (double-inside)	1.54876
	3-SF (inside *25* or *9Q*): 1,2,4,5,J (0,1,2,1,3), sp (inside)	1.54167
27**	4-F: 1,2,6,T,Q (0,1,1,1,2)	1.54167
28**	4-S: 456–89T: 1,4,5,6,9 (0,1,2,3,4)	1.54167
29**	4-S: if 345+J,Q or 9TJ+2,3,4	1.45833
30	6, 7, 8	
	2-LP: highest among 4-T: 1,2,3,8,Q (0,1,2,3,4), sp (double-inside)	1.49520
	2-LP: highest among 4-T: 1,2,3,8,J (0,1,2,3,4), sp (inside)	1.48956
	2-LP: 1,2,3,8,T (0,1,2,3,4), sp (outside)	1.48410
	2-LP: 1,2,3,8,9 (0,1,2,3,4), sp (outside1)	1.47855
	2-LP: 1,2,8,J,Q (0,1,2,3,4), 2sp (double-inside+inside)	1.48271
	2-LP: 1,2,8,T,Q (0,1,2,3,4), 2sp (double-inside+outside)	1.47716
	2-LP: 1,2,8,9,Q (0,1,2,3,4), 2sp (double-inside+outside1)	1.47161
	2-LP: 1,2,8,T,J (0,1,2,3,4), 2sp (inside+outside)	1.47231
	2-LP: 1,2,8,9,J (0,1,2,3,4), 2sp (inside+outside.1)	1.46676
	2-LP: 1,2,4,8,Q (0,1,2,3,4), (2sp) (double-inside)	1.48688
	2-LP: 1,2,5,8,Q (0,1,2,3,4), (2sp) (double-inside+inside)	1.48133
	2-LP: 1,2,6,8,Q (0,1,2,3,4), (2sp) (double-inside+outside)	1.47577
	2-LP: 1,2,5,8,J (0,1,2,3,4), (2sp) (inside)	1.47578

Rank	**Description**	**E[R]**
31	T	
	2-LP: 1,2,3,5,T (0,1,2,3,4)	1.49324
	2-LP: 1,2,3,6,T (0,1,2,3,4), sp (double-inside)	1.48491
	2-LP: 1,2,3,7,T (0,1,2,3,4), sp (inside)	1.47936
	2-LP: 1,2,3,8,T (0,1,2,3,4), sp (outside)	1.48410
	2-LP: 1,2,4,9,T (0,1,2,3,4), sp (outside1)	1.47011
	2-LP: 1,2,3,T,Q (0,1,2,3,4) sp (outside.h)	1.47751
	2-LP: 1,2,3,T,J (0,1,2,3,4) sp (outside1.h)	1.47196
	2-LP: 1,2,6,7,T (0,1,2,3,4) 2sp (double-inside+inside)	1.47242
32	5, 9	
	2-LP: 1,5,T,J,Q (0,1,2,3,4)	1.49260
	2-LP: 1,5,9,J,Q (0,1,2,3,4), sp (double-inside)	1.48427
	2-LP: 1,5,8,J,Q (0,1,2,3,4), sp (inside)	1.47572
	2-LP: 1,2,5,J,Q (0,1,2,3,4), sp (inside)	1.48057
	2-LP: 1,3,5,J,Q (0,1,2,3,4), sp (outside)	1.47502
	2-LP: 1,4,5,J,Q (0,1,2,3,4), sp (outside1)	1.46947
	2-LP: 1,5,8,9,Q (0,1,2,3,4), 2sp (double-inside+inside)	1.47179
	2-LP: 1,5,7,9,Q (0,1,2,3,4), 2sp (double-inside+outside)	1.46623
	2-LP: 1,5,7,8,Q (0,1,2,3,4), 2sp (inside+outside)	1.46138
	2-LP: 1,2,3,5,Q (0,1,2,3,4), 2sp (inside+outside)	1.46485
	2-LP: 1,2,4,5,Q (0,1,2,3,4), 2sp (inside+outside1)	1.45930
	2-LP: 1,2,5,9,Q (0,1,2,3,4), (2sp) (double-inside+inside)	1.47225
	2-LP: 1,3,5,9,Q (0,1,2,3,4), (2sp) (double-inside+outside)	1.46670
33	J	
	2-LP: 1,2,3,4,J (0,1,2,3,4)	1.45265
	2-LP: 1,3,4,J,Q (0,1,2,3,4)	1.43692
34**	2-LP: 4: if QJ42: 1,2,4,J,Q (0,1,2,3,4)	1.43999

Bibliography

[1] Dancer, B., Daily, L.W. (2004). *A Winner's Guide to Jacks or Better Poker*. Second edition. Las Vegas: Compton Dancer Consulting Inc.

[2] Dancer, B., Daily, L.W. (2005). *A Winner's Guide to Double Bonus Poker*. Second edition. Las Vegas: Compton Dancer Consulting Inc.

[3] Ethier, S. N. (2010). *The Doctrine of Chances: Probabilistic Aspects of Gambling.* New York: Springer.

[4] Fey, M. (2002). *Slot Machines* : *America's Favorite Gaming Device*. Sixth edition. Reno: Liberty Belle Books. 143–154.

[5] Marshall, M. (2006). *The Poker Code* : *The Correct Draw for Every Video Poker Hand of Deuces Wild*. Marshall House, Inc.

[6] Paymar, D. (2004). *Video Poker – Optimum Play*. Second edition. Pittsburgh: ConJelCO LLC.

[7] Tamburin, H. (2010, September 1). Cheet Sheets: Video poker strategy cards are perfectly legal - so why aren't more players using them to make the right plays? *Strictly Slots Magazine*. Retrieved from http://www.casinocenter.com/?p=1557

[8] (2010, July 8). You Bet. *The Economist*. Retrieved from http://www.economist.com/node/16539402